Rubi 4.16.0.4 Integration Test Results

on the problems in the test-suite directory "7 Inverse hyperbolic functions"

Test results for the 156 problems in "7.1.2 (d x)^m (a+b arcsinh(c x))^n.m"

Test results for the 663 problems in "7.1.4 (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n.m"

Problem 45: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\left(d+c^2\,d\,x^2\right)^2}\,dx$$

Optimal (type 4, 239 leaves, 19 steps):

$$\frac{b\,c^3}{3\,d^2\sqrt{1+c^2x^2}}-\frac{b\,c}{6\,d^2\,x^2\sqrt{1+c^2x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^2\,x^3\left(1+c^2x^2\right)}+\frac{5\,c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^2\,x\left(1+c^2x^2\right)}+\frac{5\,c^4\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d^2\left(1+c^2x^2\right)}+\frac{5\,c^3\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d^2}+\frac{13\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2x^2}\right]}{6\,d^2}-\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}+\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}$$

Result (type 4, 264 leaves, 19 steps):

$$\frac{5\,b\,c^3}{6\,d^2\sqrt{1+c^2x^2}}+\frac{b\,c}{3\,d^2\,x^2\sqrt{1+c^2x^2}}-\frac{b\,c\sqrt{1+c^2x^2}}{2\,d^2\,x^2}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^2\,x^3\left(1+c^2x^2\right)}+\frac{5\,c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^2\,x\left(1+c^2x^2\right)}+\frac{5\,c^4\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d^2\left(1+c^2x^2\right)}+\frac{5\,c^3\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d^2}+\frac{13\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2x^2}\right]}{6\,d^2}-\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}+\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}$$

Problem 54: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\left(d+c^2\,d\,x^2\right)^3}\,dx$$

Optimal (type 4, 295 leaves, 23 steps):

$$-\frac{b\,c^3}{12\,d^3\left(1+c^2\,x^2\right)^{3/2}}-\frac{b\,c}{6\,d^3\,x^2\left(1+c^2\,x^2\right)^{3/2}}+\frac{29\,b\,c^3}{24\,d^3\sqrt{1+c^2\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^3\,x^3\left(1+c^2\,x^2\right)^2}+\frac{7\,c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^3\,x\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)}{12\,d^3\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,d^3\left(1+c^2\,x^2\right)}+\frac{35\,c^3\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{4\,d^3}+\frac{19\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2\,x^2}\right]}{6\,d^3}-\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}+\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}$$

Result (type 4, 345 leaves, 23 steps):

$$\frac{7\,b\,c^3}{36\,d^3\left(1+c^2\,x^2\right)^{3/2}}+\frac{b\,c}{9\,d^3\,x^2\left(1+c^2\,x^2\right)^{3/2}}+\frac{49\,b\,c^3}{24\,d^3\sqrt{1+c^2\,x^2}}+\frac{5\,b\,c}{9\,d^3\,x^2\sqrt{1+c^2\,x^2}}-\frac{5\,b\,c\sqrt{1+c^2\,x^2}}{6\,d^3\,x^2}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^3\,x^3\left(1+c^2\,x^2\right)^2}+\frac{7\,c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^3\,x\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)}{12\,d^3\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,d^3\left(1+c^2\,x^2\right)}+\frac{35\,c^3\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{4\,d^3}+\frac{19\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2\,x^2}\right]}{6\,d^3}-\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}+\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}$$

Problem 56: Result valid but suboptimal antiderivative.

$$\int x^2\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 119 leaves, 5 steps):

$$-\frac{b\sqrt{\pi}\,x^2}{16\,c}-\frac{1}{16}\,b\,c\sqrt{\pi}\,x^4+\frac{\sqrt{\pi}\,x\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c^2}+\frac{1}{4}\,x^3\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{\sqrt{\pi}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c^3}$$

Result (type 3, 181 leaves, 5 steps):

$$-\frac{b\,x^2\sqrt{\pi+c^2\,\pi\,x^2}}{16\,c\sqrt{1+c^2\,x^2}}-\frac{b\,c\,x^4\sqrt{\pi+c^2\,\pi\,x^2}}{16\sqrt{1+c^2\,x^2}}+\frac{x\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c^2}+\frac{1}{4}\,x^3\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c^3\sqrt{1+c^2\,x^2}}$$

Problem 57: Result valid but suboptimal antiderivative.

$$\int x \sqrt{\pi + c^2 \pi x^2} \, (a + b \,\text{ArcSinh}[c\, x]) \, dx$$

Optimal (type 3, 61 leaves, 2 steps):

$$-\frac{b \sqrt{\pi}\, x}{3 c} - \frac{1}{9} b\, c \sqrt{\pi}\, x^3 + \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} (a + b \,\text{ArcSinh}[c\, x])}{3 c^2 \pi}$$

Result (type 3, 105 leaves, 2 steps):

$$-\frac{b\, x \sqrt{\pi + c^2 \pi x^2}}{3 c \sqrt{1 + c^2 x^2}} - \frac{b\, c\, x^3 \sqrt{\pi + c^2 \pi x^2}}{9 \sqrt{1 + c^2 x^2}} + \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} (a + b \,\text{ArcSinh}[c\, x])}{3 c^2 \pi}$$

Problem 58: Result valid but suboptimal antiderivative.

$$\int \sqrt{\pi + c^2 \pi x^2} \, (a + b \,\text{ArcSinh}[c\, x]) \, dx$$

Optimal (type 3, 67 leaves, 3 steps):

$$-\frac{1}{4} b\, c \sqrt{\pi}\, x^2 + \frac{1}{2} x \sqrt{\pi + c^2 \pi x^2} \, (a + b \,\text{ArcSinh}[c\, x]) + \frac{\sqrt{\pi} \, (a + b \,\text{ArcSinh}[c\, x])^2}{4 b\, c}$$

Result (type 3, 111 leaves, 3 steps):

$$-\frac{b\, c\, x^2 \sqrt{\pi + c^2 \pi x^2}}{4 \sqrt{1 + c^2 x^2}} + \frac{1}{2} x \sqrt{\pi + c^2 \pi x^2} \, (a + b \,\text{ArcSinh}[c\, x]) + \frac{\sqrt{\pi + c^2 \pi x^2} \, (a + b \,\text{ArcSinh}[c\, x])^2}{4 b\, c \sqrt{1 + c^2 x^2}}$$

Problem 59: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{\pi + c^2 \pi x^2} \, (a + b \,\text{ArcSinh}[c\, x])}{x} \, dx$$

Optimal (type 4, 89 leaves, 8 steps):

$$-b\, c \sqrt{\pi}\, x + \sqrt{\pi + c^2 \pi x^2} \, (a + b \,\text{ArcSinh}[c\, x]) -$$
$$2 \sqrt{\pi} \, (a + b \,\text{ArcSinh}[c\, x]) \,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\, x]}\right] - b \sqrt{\pi} \,\text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\, x]}\right] + b \sqrt{\pi} \,\text{PolyLog}\left[2, e^{\text{ArcSinh}[c\, x]}\right]$$

Result (type 4, 177 leaves, 8 steps):

$$-\frac{b\,c\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}{\sqrt{1+c^2\,x^2}}+\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-\frac{b\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}+\frac{b\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}$$

Problem 60: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^2}\,dx$$

Optimal (type 3, 61 leaves, 3 steps):

$$-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}+\frac{c\,\sqrt{\pi}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b}+b\,c\,\sqrt{\pi}\,\text{Log}[x]$$

Result (type 3, 105 leaves, 3 steps):

$$-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}+\frac{c\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,\sqrt{1+c^2\,x^2}}+\frac{b\,c\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{Log}[x]}{\sqrt{1+c^2\,x^2}}$$

Problem 61: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^3}\,dx$$

Optimal (type 4, 113 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{\pi}}{2\,x}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,x^2}-c^2\,\sqrt{\pi}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]-\frac{1}{2}\,b\,c^2\,\sqrt{\pi}\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]+\frac{1}{2}\,b\,c^2\,\sqrt{\pi}\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]$$

Result (type 4, 201 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{\pi+c^2\,\pi\,x^2}}{2\,x\,\sqrt{1+c^2\,x^2}}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,x^2}-\frac{c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-\frac{b\,c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{1+c^2\,x^2}}+\frac{b\,c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{1+c^2\,x^2}}$$

Problem 62: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{\pi + c^2 \pi x^2}\ (a + b\,\text{ArcSinh}[c\,x])}{x^4}\,dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$-\frac{b\,c\,\sqrt{\pi}}{6\,x^2} - \frac{(\pi + c^2 \pi x^2)^{3/2}\ (a + b\,\text{ArcSinh}[c\,x])}{3\,\pi\,x^3} + \frac{1}{3}\,b\,c^3\,\sqrt{\pi}\ \text{Log}[x]$$

Result (type 3, 106 leaves, 3 steps):

$$-\frac{b\,c\,\sqrt{\pi + c^2 \pi x^2}}{6\,x^2\,\sqrt{1 + c^2 x^2}} - \frac{(\pi + c^2 \pi x^2)^{3/2}\ (a + b\,\text{ArcSinh}[c\,x])}{3\,\pi\,x^3} + \frac{b\,c^3\,\sqrt{\pi + c^2 \pi x^2}\ \text{Log}[x]}{3\,\sqrt{1 + c^2 x^2}}$$

Problem 64: Result valid but suboptimal antiderivative.

$$\int x^2\,(\pi + c^2 \pi x^2)^{3/2}\ (a + b\,\text{ArcSinh}[c\,x])\,dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{b\,\pi^{3/2}\,x^2}{32\,c} - \frac{7}{96}\,b\,c\,\pi^{3/2}\,x^4 - \frac{1}{36}\,b\,c^3\,\pi^{3/2}\,x^6 + \frac{\pi^{3/2}\,x\,\sqrt{1 + c^2 x^2}\ (a + b\,\text{ArcSinh}[c\,x])}{16\,c^2} +$$
$$\frac{1}{8}\,\pi\,x^3\,\sqrt{\pi + c^2 \pi x^2}\ (a + b\,\text{ArcSinh}[c\,x]) + \frac{1}{6}\,x^3\,(\pi + c^2 \pi x^2)^{3/2}\ (a + b\,\text{ArcSinh}[c\,x]) - \frac{\pi^{3/2}\,(a + b\,\text{ArcSinh}[c\,x])^2}{32\,b\,c^3}$$

Result (type 3, 254 leaves, 8 steps):

$$-\frac{b\,\pi\,x^2\,\sqrt{\pi + c^2 \pi x^2}}{32\,c\,\sqrt{1 + c^2 x^2}} - \frac{7\,b\,c\,\pi\,x^4\,\sqrt{\pi + c^2 \pi x^2}}{96\,\sqrt{1 + c^2 x^2}} - \frac{b\,c^3\,\pi\,x^6\,\sqrt{\pi + c^2 \pi x^2}}{36\,\sqrt{1 + c^2 x^2}} + \frac{\pi\,x\,\sqrt{\pi + c^2 \pi x^2}\ (a + b\,\text{ArcSinh}[c\,x])}{16\,c^2} +$$
$$\frac{1}{8}\,\pi\,x^3\,\sqrt{\pi + c^2 \pi x^2}\ (a + b\,\text{ArcSinh}[c\,x]) + \frac{1}{6}\,x^3\,(\pi + c^2 \pi x^2)^{3/2}\ (a + b\,\text{ArcSinh}[c\,x]) - \frac{\pi\,\sqrt{\pi + c^2 \pi x^2}\ (a + b\,\text{ArcSinh}[c\,x])^2}{32\,b\,c^3\,\sqrt{1 + c^2 x^2}}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x\,(\pi + c^2 \pi x^2)^{3/2}\ (a + b\,\text{ArcSinh}[c\,x])\,dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{b\,\pi^{3/2}\,x}{5\,c}-\frac{2}{15}\,b\,c\,\pi^{3/2}\,x^3-\frac{1}{25}\,b\,c^3\,\pi^{3/2}\,x^5+\frac{\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{5\,c^2\,\pi}$$

Result (type 3, 146 leaves, 3 steps):

$$-\frac{b\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}{5\,c\,\sqrt{1+c^2\,x^2}}-\frac{2\,b\,c\,\pi\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}}{15\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\pi\,x^5\,\sqrt{\pi+c^2\,\pi\,x^2}}{25\,\sqrt{1+c^2\,x^2}}+\frac{\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{5\,c^2\,\pi}$$

Problem 66: Result valid but suboptimal antiderivative.

$$\int\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 111 leaves, 6 steps):

$$-\frac{5}{16}\,b\,c\,\pi^{3/2}\,x^2-\frac{1}{16}\,b\,c^3\,\pi^{3/2}\,x^4+\frac{3}{8}\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{4}\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{3\,\pi^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c}$$

Result (type 3, 180 leaves, 6 steps):

$$-\frac{5\,b\,c\,\pi\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}{16\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\pi\,x^4\,\sqrt{\pi+c^2\,\pi\,x^2}}{16\,\sqrt{1+c^2\,x^2}}+\frac{3}{8}\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+$$
$$\frac{1}{4}\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{3\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c\,\sqrt{1+c^2\,x^2}}$$

Problem 67: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}\,dx$$

Optimal (type 4, 134 leaves, 10 steps):

$$-\frac{4}{3}\,b\,c\,\pi^{3/2}\,x-\frac{1}{9}\,b\,c^3\,\pi^{3/2}\,x^3+\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{3}\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$2\,\pi^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]-b\,\pi^{3/2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]+b\,\pi^{3/2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]$$

Result (type 4, 249 leaves, 10 steps):

$$-\frac{4\,b\,c\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}{3\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\pi\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}}{9\,\sqrt{1+c^2\,x^2}}+\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{3}\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$\frac{2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-\frac{b\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}+\frac{b\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}$$

Problem 68: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{x^2} \, dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$-\frac{1}{4} b c^3 \pi^{3/2} x^2 + \frac{3}{2} c^2 \pi x \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{x} + \frac{3 c \pi^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2}{4 b} + b c \pi^{3/2} \operatorname{Log}[x]$$

Result (type 3, 177 leaves, 6 steps):

$$-\frac{b c^3 \pi x^2 \sqrt{\pi + c^2 \pi x^2}}{4 \sqrt{1 + c^2 x^2}} + \frac{3}{2} c^2 \pi x \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{x} + \frac{3 c \pi \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2}{4 b \sqrt{1 + c^2 x^2}} + \frac{b c \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{Log}[x]}{\sqrt{1 + c^2 x^2}}$$

Problem 69: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{x^3} \, dx$$

Optimal (type 4, 155 leaves, 11 steps):

$$-\frac{b c \pi^{3/2}}{2 x} - b c^3 \pi^{3/2} x + \frac{3}{2} c^2 \pi \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{2 x^2} - 3 c^2 \pi^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right) \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right] - \frac{3}{2} b c^2 \pi^{3/2} \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right] + \frac{3}{2} b c^2 \pi^{3/2} \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]$$

Result (type 4, 270 leaves, 11 steps):

$$-\frac{b c \pi \sqrt{\pi + c^2 \pi x^2}}{2 x \sqrt{1 + c^2 x^2}} - \frac{b c^3 \pi x \sqrt{\pi + c^2 \pi x^2}}{\sqrt{1 + c^2 x^2}} + \frac{3}{2} c^2 \pi \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{2 x^2} - \frac{3 c^2 \pi \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right) \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{1 + c^2 x^2}} - \frac{3 b c^2 \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{2 \sqrt{1 + c^2 x^2}} + \frac{3 b c^2 \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{2 \sqrt{1 + c^2 x^2}}$$

Problem 70: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} (a + b \,\text{ArcSinh}[c\, x])}{x^4} \, dx$$

Optimal (type 3, 115 leaves, 6 steps):

$$-\frac{b\, c\, \pi^{3/2}}{6 x^2} - \frac{c^2 \pi \sqrt{\pi + c^2 \pi x^2} \,(a + b\, \text{ArcSinh}[c\, x])}{x} - \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} (a + b\, \text{ArcSinh}[c\, x])}{3 x^3} + \frac{c^3 \pi^{3/2} (a + b\, \text{ArcSinh}[c\, x])^2}{2 b} + \frac{4}{3} b\, c^3 \pi^{3/2} \,\text{Log}[x]$$

Result (type 3, 184 leaves, 6 steps):

$$-\frac{b\, c\, \pi \sqrt{\pi + c^2 \pi x^2}}{6 x^2 \sqrt{1 + c^2 x^2}} - \frac{c^2 \pi \sqrt{\pi + c^2 \pi x^2} \,(a + b\, \text{ArcSinh}[c\, x])}{x} - \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} (a + b\, \text{ArcSinh}[c\, x])}{3 x^3} + \frac{c^3 \pi \sqrt{\pi + c^2 \pi x^2} \,(a + b\, \text{ArcSinh}[c\, x])^2}{2 b \sqrt{1 + c^2 x^2}} + \frac{4 b\, c^3 \pi \sqrt{\pi + c^2 \pi x^2} \,\text{Log}[x]}{3 \sqrt{1 + c^2 x^2}}$$

Problem 72: Result valid but suboptimal antiderivative.

$$\int x^2 \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b\, \text{ArcSinh}[c\, x]) \, dx$$

Optimal (type 3, 213 leaves, 12 steps):

$$-\frac{5 b\, \pi^{5/2} x^2}{256 c} - \frac{59}{768} b\, c\, \pi^{5/2} x^4 - \frac{17}{288} b\, c^3 \pi^{5/2} x^6 - \frac{1}{64} b\, c^5 \pi^{5/2} x^8 + \frac{5 \pi^{5/2} x \sqrt{1 + c^2 x^2} \,(a + b\, \text{ArcSinh}[c\, x])}{128 c^2} + \frac{5}{64} \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2} \,(a + b\, \text{ArcSinh}[c\, x]) + \frac{5}{48} \pi x^3 \left(\pi + c^2 \pi x^2\right)^{3/2} (a + b\, \text{ArcSinh}[c\, x]) + \frac{1}{8} x^3 \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b\, \text{ArcSinh}[c\, x]) - \frac{5 \pi^{5/2} (a + b\, \text{ArcSinh}[c\, x])^2}{256 b\, c^3}$$

Result (type 3, 337 leaves, 12 steps):

$$-\frac{5 b\, \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2}}{256 c \sqrt{1 + c^2 x^2}} - \frac{59 b\, c\, \pi^2 x^4 \sqrt{\pi + c^2 \pi x^2}}{768 \sqrt{1 + c^2 x^2}} - \frac{17 b\, c^3 \pi^2 x^6 \sqrt{\pi + c^2 \pi x^2}}{288 \sqrt{1 + c^2 x^2}} - \frac{b\, c^5 \pi^2 x^8 \sqrt{\pi + c^2 \pi x^2}}{64 \sqrt{1 + c^2 x^2}} + \frac{5 \pi^2 x \sqrt{\pi + c^2 \pi x^2} \,(a + b\, \text{ArcSinh}[c\, x])}{128 c^2} + \frac{5}{64} \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2} \,(a + b\, \text{ArcSinh}[c\, x]) + \frac{5}{48} \pi x^3 \left(\pi + c^2 \pi x^2\right)^{3/2} (a + b\, \text{ArcSinh}[c\, x]) + \frac{1}{8} x^3 \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b\, \text{ArcSinh}[c\, x]) - \frac{5 \pi^2 \sqrt{\pi + c^2 \pi x^2} \,(a + b\, \text{ArcSinh}[c\, x])^2}{256 b\, c^3 \sqrt{1 + c^2 x^2}}$$

Problem 73: Result valid but suboptimal antiderivative.

$$\int x \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b \operatorname{ArcSinh}[c x]) \, dx$$

Optimal (type 3, 93 leaves, 3 steps):

$$-\frac{b \pi^{5/2} x}{7 c} - \frac{1}{7} b c \pi^{5/2} x^3 - \frac{3}{35} b c^3 \pi^{5/2} x^5 - \frac{1}{49} b c^5 \pi^{5/2} x^7 + \frac{\left(\pi + c^2 \pi x^2\right)^{7/2} (a + b \operatorname{ArcSinh}[c x])}{7 c^2 \pi}$$

Result (type 3, 193 leaves, 3 steps):

$$-\frac{b \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{7 c \sqrt{1 + c^2 x^2}} - \frac{b c \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2}}{7 \sqrt{1 + c^2 x^2}} - \frac{3 b c^3 \pi^2 x^5 \sqrt{\pi + c^2 \pi x^2}}{35 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^7 \sqrt{\pi + c^2 \pi x^2}}{49 \sqrt{1 + c^2 x^2}} + \frac{\left(\pi + c^2 \pi x^2\right)^{7/2} (a + b \operatorname{ArcSinh}[c x])}{7 c^2 \pi}$$

Problem 74: Result valid but suboptimal antiderivative.

$$\int \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b \operatorname{ArcSinh}[c x]) \, dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{25}{96} b c \pi^{5/2} x^2 - \frac{5}{96} b c^3 \pi^{5/2} x^4 - \frac{b \pi^{5/2} \left(1 + c^2 x^2\right)^3}{36 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5}{24} \pi x \left(\pi + c^2 \pi x^2\right)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} x \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5 \pi^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{32 b c}$$

Result (type 3, 254 leaves, 8 steps):

$$-\frac{25 b c \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2}}{96 \sqrt{1 + c^2 x^2}} - \frac{5 b c^3 \pi^2 x^4 \sqrt{\pi + c^2 \pi x^2}}{96 \sqrt{1 + c^2 x^2}} - \frac{b \pi^2 \left(1 + c^2 x^2\right)^{5/2} \sqrt{\pi + c^2 \pi x^2}}{36 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5}{24} \pi x \left(\pi + c^2 \pi x^2\right)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} x \left(\pi + c^2 \pi x^2\right)^{5/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{32 b c \sqrt{1 + c^2 x^2}}$$

Problem 75: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x} \, dx$$

Optimal (type 4, 179 leaves, 13 steps):

$$-\frac{23}{15} b c \pi^{5/2} x-\frac{11}{45} b c^3 \pi^{5/2} x^3-\frac{1}{25} b c^5 \pi^{5/2} x^5+\pi^2 \sqrt{\pi+c^2 \pi x^2} (a+b \text{ArcSinh}[c x])+$$
$$\frac{1}{3} \pi (\pi+c^2 \pi x^2)^{3/2} (a+b \text{ArcSinh}[c x])+\frac{1}{5} (\pi+c^2 \pi x^2)^{5/2} (a+b \text{ArcSinh}[c x])-$$
$$2 \pi^{5/2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]-b \pi^{5/2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]+b \pi^{5/2} \text{PolyLog}\left[2, e^{\text{ArcSinh}[c x]}\right]$$

Result (type 4, 329 leaves, 13 steps):

$$-\frac{23 b c \pi^2 x \sqrt{\pi+c^2 \pi x^2}}{15 \sqrt{1+c^2 x^2}}-\frac{11 b c^3 \pi^2 x^3 \sqrt{\pi+c^2 \pi x^2}}{45 \sqrt{1+c^2 x^2}}-\frac{b c^5 \pi^2 x^5 \sqrt{\pi+c^2 \pi x^2}}{25 \sqrt{1+c^2 x^2}}+\pi^2 \sqrt{\pi+c^2 \pi x^2} (a+b \text{ArcSinh}[c x])+$$
$$\frac{1}{3} \pi (\pi+c^2 \pi x^2)^{3/2} (a+b \text{ArcSinh}[c x])+\frac{1}{5} (\pi+c^2 \pi x^2)^{5/2} (a+b \text{ArcSinh}[c x])-\frac{2 \pi^2 \sqrt{\pi+c^2 \pi x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{\sqrt{1+c^2 x^2}}-$$
$$\frac{b \pi^2 \sqrt{\pi+c^2 \pi x^2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{\sqrt{1+c^2 x^2}}+\frac{b \pi^2 \sqrt{\pi+c^2 \pi x^2} \text{PolyLog}\left[2, e^{\text{ArcSinh}[c x]}\right]}{\sqrt{1+c^2 x^2}}$$

Problem 76: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi+c^2 \pi x^2)^{5/2} (a+b \text{ArcSinh}[c x])}{x^2} dx$$

Optimal (type 3, 157 leaves, 10 steps):

$$-\frac{9}{16} b c^3 \pi^{5/2} x^2-\frac{1}{16} b c^5 \pi^{5/2} x^4+\frac{15}{8} c^2 \pi^2 x \sqrt{\pi+c^2 \pi x^2} (a+b \text{ArcSinh}[c x])+$$
$$\frac{5}{4} c^2 \pi x (\pi+c^2 \pi x^2)^{3/2} (a+b \text{ArcSinh}[c x])-\frac{(\pi+c^2 \pi x^2)^{5/2} (a+b \text{ArcSinh}[c x])}{x}+\frac{15 c \pi^{5/2} (a+b \text{ArcSinh}[c x])^2}{16 b}+b c \pi^{5/2} \text{Log}[x]$$

Result (type 3, 257 leaves, 10 steps):

$$-\frac{9 b c^3 \pi^2 x^2 \sqrt{\pi+c^2 \pi x^2}}{16 \sqrt{1+c^2 x^2}}-\frac{b c^5 \pi^2 x^4 \sqrt{\pi+c^2 \pi x^2}}{16 \sqrt{1+c^2 x^2}}+\frac{15}{8} c^2 \pi^2 x \sqrt{\pi+c^2 \pi x^2} (a+b \text{ArcSinh}[c x])+\frac{5}{4} c^2 \pi x (\pi+c^2 \pi x^2)^{3/2} (a+b \text{ArcSinh}[c x])-$$
$$\frac{(\pi+c^2 \pi x^2)^{5/2} (a+b \text{ArcSinh}[c x])}{x}+\frac{15 c \pi^2 \sqrt{\pi+c^2 \pi x^2} (a+b \text{ArcSinh}[c x])^2}{16 b \sqrt{1+c^2 x^2}}+\frac{b c \pi^2 \sqrt{\pi+c^2 \pi x^2} \text{Log}[x]}{\sqrt{1+c^2 x^2}}$$

Problem 77: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi+c^2 \pi x^2)^{5/2} (a+b \text{ArcSinh}[c x])}{x^3} dx$$

Optimal (type 4, 205 leaves, 13 steps):

$$-\frac{b c \pi^{5/2}}{2 x}-\frac{7}{3} b c^3 \pi^{5/2} x-\frac{1}{9} b c^5 \pi^{5/2} x^3+\frac{5}{2} c^2 \pi^2 \sqrt{\pi+c^2 \pi x^2}\,(a+b\,\text{ArcSinh}[c x])+\frac{5}{6} c^2 \pi\,(\pi+c^2 \pi x^2)^{3/2}\,(a+b\,\text{ArcSinh}[c x])-\frac{(\pi+c^2 \pi x^2)^{5/2}\,(a+b\,\text{ArcSinh}[c x])}{2 x^2}-5 c^2 \pi^{5/2}\,(a+b\,\text{ArcSinh}[c x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]-\frac{5}{2} b c^2 \pi^{5/2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]+\frac{5}{2} b c^2 \pi^{5/2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]$$

Result (type 4, 355 leaves, 13 steps):

$$-\frac{b c \pi^2 \sqrt{\pi+c^2 \pi x^2}}{2 x \sqrt{1+c^2 x^2}}-\frac{7 b c^3 \pi^2 x \sqrt{\pi+c^2 \pi x^2}}{3 \sqrt{1+c^2 x^2}}-\frac{b c^5 \pi^2 x^3 \sqrt{\pi+c^2 \pi x^2}}{9 \sqrt{1+c^2 x^2}}+\frac{5}{2} c^2 \pi^2 \sqrt{\pi+c^2 \pi x^2}\,(a+b\,\text{ArcSinh}[c x])+\frac{5}{6} c^2 \pi\,(\pi+c^2 \pi x^2)^{3/2}\,(a+b\,\text{ArcSinh}[c x])-\frac{(\pi+c^2 \pi x^2)^{5/2}\,(a+b\,\text{ArcSinh}[c x])}{2 x^2}-\frac{5 c^2 \pi^2 \sqrt{\pi+c^2 \pi x^2}\,(a+b\,\text{ArcSinh}[c x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{\sqrt{1+c^2 x^2}}-\frac{5 b c^2 \pi^2 \sqrt{\pi+c^2 \pi x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{2 \sqrt{1+c^2 x^2}}+\frac{5 b c^2 \pi^2 \sqrt{\pi+c^2 \pi x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{2 \sqrt{1+c^2 x^2}}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi+c^2 \pi x^2)^{5/2}\,(a+b\,\text{ArcSinh}[c x])}{x^4}\,dx$$

Optimal (type 3, 166 leaves, 10 steps):

$$-\frac{b c \pi^{5/2}}{6 x^2}-\frac{1}{4} b c^5 \pi^{5/2} x^2+\frac{5}{2} c^4 \pi^2 x \sqrt{\pi+c^2 \pi x^2}\,(a+b\,\text{ArcSinh}[c x])-\frac{5 c^2 \pi\,(\pi+c^2 \pi x^2)^{3/2}\,(a+b\,\text{ArcSinh}[c x])}{3 x}-\frac{(\pi+c^2 \pi x^2)^{5/2}\,(a+b\,\text{ArcSinh}[c x])}{3 x^3}+\frac{5 c^3 \pi^{5/2}\,(a+b\,\text{ArcSinh}[c x])^2}{4 b}+\frac{7}{3} b c^3 \pi^{5/2}\,\text{Log}[x]$$

Result (type 3, 266 leaves, 10 steps):

$$-\frac{b c \pi^2 \sqrt{\pi+c^2 \pi x^2}}{6 x^2 \sqrt{1+c^2 x^2}}-\frac{b c^5 \pi^2 x^2 \sqrt{\pi+c^2 \pi x^2}}{4 \sqrt{1+c^2 x^2}}+\frac{5}{2} c^4 \pi^2 x \sqrt{\pi+c^2 \pi x^2}\,(a+b\,\text{ArcSinh}[c x])-\frac{5 c^2 \pi\,(\pi+c^2 \pi x^2)^{3/2}\,(a+b\,\text{ArcSinh}[c x])}{3 x}-\frac{(\pi+c^2 \pi x^2)^{5/2}\,(a+b\,\text{ArcSinh}[c x])}{3 x^3}+\frac{5 c^3 \pi^2 \sqrt{\pi+c^2 \pi x^2}\,(a+b\,\text{ArcSinh}[c x])^2}{4 b \sqrt{1+c^2 x^2}}+\frac{7 b c^3 \pi^2 \sqrt{\pi+c^2 \pi x^2}\,\text{Log}[x]}{3 \sqrt{1+c^2 x^2}}$$

Problem 80: Result valid but suboptimal antiderivative.

$$\int \frac{x^5\,(a+b\,\text{ArcSinh}[c x])}{\sqrt{\pi+c^2 \pi x^2}}\,dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$-\frac{8 b x}{15 c^5 \sqrt{\pi}}+\frac{4 b x^3}{45 c^3 \sqrt{\pi}}-\frac{b x^5}{25 c \sqrt{\pi}}+\frac{8 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{15 c^6 \pi}-\frac{4 x^2 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{15 c^4 \pi}+\frac{x^4 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{5 c^2 \pi}$$

Result (type 3, 215 leaves, 6 steps):

$$-\frac{8 b x \sqrt{1+c^2 x^2}}{15 c^5 \sqrt{\pi+c^2 \pi x^2}}+\frac{4 b x^3 \sqrt{1+c^2 x^2}}{45 c^3 \sqrt{\pi+c^2 \pi x^2}}-\frac{b x^5 \sqrt{1+c^2 x^2}}{25 c \sqrt{\pi+c^2 \pi x^2}}+\frac{8 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{15 c^6 \pi}-\frac{4 x^2 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{15 c^4 \pi}+\frac{x^4 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{5 c^2 \pi}$$

Problem 81: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\,(a+b \text{ArcSinh}[c x])}{\sqrt{\pi+c^2 \pi x^2}}\,dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{3 b x^2}{16 c^3 \sqrt{\pi}}-\frac{b x^4}{16 c \sqrt{\pi}}-\frac{3 x \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{8 c^4 \pi}+\frac{x^3 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{4 c^2 \pi}+\frac{3\,(a+b \text{ArcSinh}[c x])^2}{16 b c^5 \sqrt{\pi}}$$

Result (type 3, 170 leaves, 5 steps):

$$\frac{3 b x^2 \sqrt{1+c^2 x^2}}{16 c^3 \sqrt{\pi+c^2 \pi x^2}}-\frac{b x^4 \sqrt{1+c^2 x^2}}{16 c \sqrt{\pi+c^2 \pi x^2}}-\frac{3 x \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{8 c^4 \pi}+\frac{x^3 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{4 c^2 \pi}+\frac{3\,(a+b \text{ArcSinh}[c x])^2}{16 b c^5 \sqrt{\pi}}$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{x^3\,(a+b \text{ArcSinh}[c x])}{\sqrt{\pi+c^2 \pi x^2}}\,dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{2 b x}{3 c^3 \sqrt{\pi}}-\frac{b x^3}{9 c \sqrt{\pi}}-\frac{2 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{3 c^4 \pi}+\frac{x^2 \sqrt{\pi+c^2 \pi x^2}\,(a+b \text{ArcSinh}[c x])}{3 c^2 \pi}$$

Result (type 3, 142 leaves, 4 steps):

$$\frac{2\,b\,x\,\sqrt{1+c^2\,x^2}}{3\,c^3\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{b\,x^3\,\sqrt{1+c^2\,x^2}}{9\,c\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,\pi}+\frac{x^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,\pi}$$

Problem 83: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{\pi+c^2\,\pi\,x^2}}\,dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$-\frac{b\,x^2}{4\,c\,\sqrt{\pi}}+\frac{x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^2\,\pi}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{\pi}}$$

Result (type 3, 97 leaves, 3 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^2\,\pi}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{\pi}}$$

Problem 84: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{\pi+c^2\,\pi\,x^2}}\,dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$-\frac{b\,x}{c\,\sqrt{\pi}}+\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi}$$

Result (type 3, 64 leaves, 2 steps):

$$-\frac{b\,x\,\sqrt{1+c^2\,x^2}}{c\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi}$$

Problem 87: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}\,dx$$

Optimal (type 3, 41 leaves, 2 steps):

$$-\frac{\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{\pi\,x}+\frac{b\,c\,\text{Log}[x]}{\sqrt{\pi}}$$

Result (type 3, 63 leaves, 2 steps):

$$-\frac{\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{\pi\,x}+\frac{b\,c\,\sqrt{1+c^2x^2}\ \text{Log}[x]}{\sqrt{\pi+c^2\pi x^2}}$$

Problem 88: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx$$

Optimal (type 4, 115 leaves, 8 steps):

$$-\frac{b\,c}{2\sqrt{\pi}\ x}-\frac{\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi\,x^2}+\frac{c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{\pi}}+\frac{b\,c^2\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{\pi}}-\frac{b\,c^2\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{\pi}}$$

Result (type 4, 137 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{1+c^2x^2}}{2\,x\,\sqrt{\pi+c^2\pi x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi\,x^2}+\frac{c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{\pi}}+\frac{b\,c^2\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{\pi}}-\frac{b\,c^2\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{\pi}}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$-\frac{b\,c}{6\sqrt{\pi}\ x^2}-\frac{\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x^3}+\frac{2\,c^2\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x}-\frac{2\,b\,c^3\,\text{Log}[x]}{3\sqrt{\pi}}$$

Result (type 3, 141 leaves, 4 steps):

$$-\frac{b\,c\,\sqrt{1+c^2x^2}}{6\,x^2\sqrt{\pi+c^2\pi x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x^3}+\frac{2\,c^2\sqrt{\pi+c^2\pi x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x}-\frac{2\,b\,c^3\sqrt{1+c^2x^2}\ \text{Log}[x]}{3\sqrt{\pi+c^2\pi x^2}}$$

Problem 91: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\,(a+b\,\text{ArcSinh}[c\,x])}{(\pi+c^2\,\pi\,x^2)^{3/2}}\,dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{b\,x^2}{4\,c^3\,\pi^{3/2}}-\frac{x^3\,(a+b\,\text{ArcSinh}[c\,x])}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{3\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{2\,c^4\,\pi^2}-\frac{3\,(a+b\,\text{ArcSinh}[c\,x])^2}{4\,b\,c^5\,\pi^{3/2}}-\frac{b\,\text{Log}[1+c^2\,x^2]}{2\,c^5\,\pi^{3/2}}$$

Result (type 3, 181 leaves, 7 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c^3\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{x^3\,(a+b\,\text{ArcSinh}[c\,x])}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{3\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{2\,c^4\,\pi^2}-\frac{3\,(a+b\,\text{ArcSinh}[c\,x])^2}{4\,b\,c^5\,\pi^{3/2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\text{Log}[1+c^2\,x^2]}{2\,c^5\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 93: Result valid but suboptimal antiderivative.

$$\int \frac{x^2\,(a+b\,\text{ArcSinh}[c\,x])}{(\pi+c^2\,\pi\,x^2)^{3/2}}\,dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$-\frac{x\,(a+b\,\text{ArcSinh}[c\,x])}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{(a+b\,\text{ArcSinh}[c\,x])^2}{2\,b\,c^3\,\pi^{3/2}}+\frac{b\,\text{Log}[1+c^2\,x^2]}{2\,c^3\,\pi^{3/2}}$$

Result (type 3, 105 leaves, 3 steps):

$$-\frac{x\,(a+b\,\text{ArcSinh}[c\,x])}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{(a+b\,\text{ArcSinh}[c\,x])^2}{2\,b\,c^3\,\pi^{3/2}}+\frac{b\,\sqrt{1+c^2\,x^2}\,\text{Log}[1+c^2\,x^2]}{2\,c^3\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{x\,(a+b\,\text{ArcSinh}[c\,x])}{(\pi+c^2\,\pi\,x^2)^{3/2}}\,dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$-\frac{a+b\,\text{ArcSinh}[c\,x]}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{b\,\text{ArcTan}[c\,x]}{c^2\,\pi^{3/2}}$$

Result (type 3, 70 leaves, 2 steps):

$$-\frac{a+b\,\text{ArcSinh}[c\,x]}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 95: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 51 leaves, 2 steps):

$$\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{b\,\text{Log}\left[1+c^2\,x^2\right]}{2\,c\,\pi^{3/2}}$$

Result (type 3, 76 leaves, 2 steps):

$$\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,c\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 96: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 94 leaves, 8 steps):

$$\frac{a+b\,\text{ArcSinh}[c\,x]}{\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{b\,\text{ArcTan}[c\,x]}{\pi^{3/2}}-\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}-\frac{b\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{b\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}$$

Result (type 4, 119 leaves, 8 steps):

$$\frac{a+b\,\text{ArcSinh}[c\,x]}{\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-$$
$$\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}-\frac{b\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{b\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}$$

Problem 98: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 162 leaves, 11 steps):

$$-\frac{b\,c}{2\,\pi^{3/2}\,x}-\frac{3\,c^2\,(a+b\,\text{ArcSinh}[c\,x])}{2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,\pi\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{b\,c^2\,\text{ArcTan}[c\,x]}{\pi^{3/2}}+\frac{3\,c^2\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{3\,b\,c^2\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{3/2}}-\frac{3\,b\,c^2\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{3/2}}$$

Result (type 4, 212 leaves, 11 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{3\,c^2\,(a+b\,\text{ArcSinh}[c\,x])}{2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,\pi\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{3\,c^2\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{3\,b\,c^2\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{3/2}}-\frac{3\,b\,c^2\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{3/2}}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int\frac{x^6\,(a+b\,\text{ArcSinh}[c\,x])}{\left(\pi+c^2\,\pi\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 192 leaves, 11 steps):

$$-\frac{b\,x^2}{4\,c^5\,\pi^{5/2}}-\frac{b}{6\,c^7\,\pi^{5/2}\,(1+c^2\,x^2)}-\frac{x^5\,(a+b\,\text{ArcSinh}[c\,x])}{3\,c^2\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{5\,x^3\,(a+b\,\text{ArcSinh}[c\,x])}{3\,c^4\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{5\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{2\,c^6\,\pi^3}-\frac{5\,(a+b\,\text{ArcSinh}[c\,x])^2}{4\,b\,c^7\,\pi^{5/2}}-\frac{7\,b\,\text{Log}\left[1+c^2\,x^2\right]}{6\,c^7\,\pi^{5/2}}$$

Result (type 3, 256 leaves, 11 steps):

$$-\frac{b}{6\,c^7\,\pi^2\,\sqrt{1+c^2\,x^2}\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c^5\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{x^5\,(a+b\,\text{ArcSinh}[c\,x])}{3\,c^2\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{5\,x^3\,(a+b\,\text{ArcSinh}[c\,x])}{3\,c^4\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{5\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{2\,c^6\,\pi^3}-\frac{5\,(a+b\,\text{ArcSinh}[c\,x])^2}{4\,b\,c^7\,\pi^{5/2}}-\frac{7\,b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{6\,c^7\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 102: Result valid but suboptimal antiderivative.

$$\int\frac{x^4\,(a+b\,\text{ArcSinh}[c\,x])}{\left(\pi+c^2\,\pi\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$\frac{b}{6 c^5 \pi^{5/2} \left(1+c^2 x^2\right)}-\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{x \left(a+b \text{ArcSinh}[c x]\right)}{c^4 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{2 b c^5 \pi^{5/2}}+\frac{2 b \text{Log}\left[1+c^2 x^2\right]}{3 c^5 \pi^{5/2}}$$

Result (type 3, 178 leaves, 7 steps):

$$\frac{b}{6 c^5 \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}-\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{x \left(a+b \text{ArcSinh}[c x]\right)}{c^4 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{2 b c^5 \pi^{5/2}}+\frac{2 b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{3 c^5 \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 104: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a+b \text{ArcSinh}[c x]\right)}{\left(\pi+c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$-\frac{b}{6 c^3 \pi^{5/2} \left(1+c^2 x^2\right)}+\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{b \text{Log}\left[1+c^2 x^2\right]}{6 c^3 \pi^{5/2}}$$

Result (type 3, 119 leaves, 4 steps):

$$-\frac{b}{6 c^3 \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}+\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{6 c^3 \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 105: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(a+b \text{ArcSinh}[c x]\right)}{\left(\pi+c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$\frac{b x}{6 c \pi^{5/2} \left(1+c^2 x^2\right)}-\frac{a+b \text{ArcSinh}[c x]}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+\frac{b \text{ArcTan}[c x]}{6 c^2 \pi^{5/2}}$$

Result (type 3, 114 leaves, 3 steps):

$$\frac{b x}{6 c \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}-\frac{a+b \text{ArcSinh}[c x]}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+\frac{b \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{6 c^2 \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 106: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \,\text{ArcSinh}[c\, x]}{\left(\pi + c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{b}{6\, c\, \pi^{5/2} \left(1 + c^2 x^2\right)} + \frac{x \left(a + b\, \text{ArcSinh}[c\, x]\right)}{3\, \pi \left(\pi + c^2 \pi x^2\right)^{3/2}} + \frac{2\, x \left(a + b\, \text{ArcSinh}[c\, x]\right)}{3\, \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{b\, \text{Log}\left[1 + c^2 x^2\right]}{3\, c\, \pi^{5/2}}$$

Result (type 3, 147 leaves, 4 steps):

$$\frac{b}{6\, c\, \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} + \frac{x \left(a + b\, \text{ArcSinh}[c\, x]\right)}{3\, \pi \left(\pi + c^2 \pi x^2\right)^{3/2}} + \frac{2\, x \left(a + b\, \text{ArcSinh}[c\, x]\right)}{3\, \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{b \sqrt{1 + c^2 x^2}\, \text{Log}\left[1 + c^2 x^2\right]}{3\, c\, \pi^2 \sqrt{\pi + c^2 \pi x^2}}$$

Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \,\text{ArcSinh}[c\, x]}{x \left(\pi + c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 148 leaves, 11 steps):

$$-\frac{b\, c\, x}{6\, \pi^{5/2} \left(1 + c^2 x^2\right)} + \frac{a + b\, \text{ArcSinh}[c\, x]}{3\, \pi \left(\pi + c^2 \pi x^2\right)^{3/2}} + \frac{a + b\, \text{ArcSinh}[c\, x]}{\pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{7\, b\, \text{ArcTan}[c\, x]}{6\, \pi^{5/2}} - \frac{2 \left(a + b\, \text{ArcSinh}[c\, x]\right) \text{ArcTanh}\left[e^{\text{ArcSinh}[c\, x]}\right]}{\pi^{5/2}} - \frac{b\, \text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\, x]}\right]}{\pi^{5/2}} + \frac{b\, \text{PolyLog}\left[2, e^{\text{ArcSinh}[c\, x]}\right]}{\pi^{5/2}}$$

Result (type 4, 187 leaves, 11 steps):

$$-\frac{b\, c\, x}{6\, \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} + \frac{a + b\, \text{ArcSinh}[c\, x]}{3\, \pi \left(\pi + c^2 \pi x^2\right)^{3/2}} + \frac{a + b\, \text{ArcSinh}[c\, x]}{\pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{7\, b \sqrt{1 + c^2 x^2}\, \text{ArcTan}[c\, x]}{6\, \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{2 \left(a + b\, \text{ArcSinh}[c\, x]\right) \text{ArcTanh}\left[e^{\text{ArcSinh}[c\, x]}\right]}{\pi^{5/2}} - \frac{b\, \text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\, x]}\right]}{\pi^{5/2}} + \frac{b\, \text{PolyLog}\left[2, e^{\text{ArcSinh}[c\, x]}\right]}{\pi^{5/2}}$$

Problem 109: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \,\text{ArcSinh}[c\, x]}{x^3 \left(\pi + c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 247 leaves, 15 steps):

$$-\frac{3\,b\,c}{4\,\pi^{5/2}\,x}+\frac{b\,c}{4\,\pi^{5/2}\,x\,\left(1+c^2\,x^2\right)}+\frac{5\,b\,c^3\,x}{12\,\pi^{5/2}\,\left(1+c^2\,x^2\right)}-\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{6\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,\pi\,x^2\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+$$
$$\frac{13\,b\,c^2\,\text{ArcTan}[c\,x]}{6\,\pi^{5/2}}+\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}+\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}-\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}$$

Result (type 4, 325 leaves, 15 steps):

$$\frac{b\,c}{4\,\pi^2\,x\,\sqrt{1+c^2\,x^2}\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{5\,b\,c^3\,x}{12\,\pi^2\,\sqrt{1+c^2\,x^2}\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{3\,b\,c\,\sqrt{1+c^2\,x^2}}{4\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}-$$
$$\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{6\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,\pi\,x^2\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{13\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{6\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+$$
$$\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}+\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}-\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}$$

Problem 120: Result optimal but 3 more steps used.

$$\int x^3\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 175 leaves, 3 steps):

$$\frac{2\,b\,x\,\sqrt{d+c^2\,d\,x^2}}{15\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,x^3\,\sqrt{d+c^2\,d\,x^2}}{45\,c\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,x^5\,\sqrt{d+c^2\,d\,x^2}}{25\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{5\,c^4\,d^2}$$

Result (type 3, 175 leaves, 6 steps):

$$\frac{2\,b\,x\,\sqrt{d+c^2\,d\,x^2}}{15\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,x^3\,\sqrt{d+c^2\,d\,x^2}}{45\,c\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,x^5\,\sqrt{d+c^2\,d\,x^2}}{25\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{5\,c^4\,d^2}$$

Problem 128: Result optimal but 3 more steps used.

$$\int x^3\,\left(d+c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 217 leaves, 4 steps):

$$\frac{2\,b\,d\,x\,\sqrt{d+c^2\,d\,x^2}}{35\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,d\,x^3\,\sqrt{d+c^2\,d\,x^2}}{105\,c\,\sqrt{1+c^2\,x^2}}-\frac{8\,b\,c\,d\,x^5\,\sqrt{d+c^2\,d\,x^2}}{175\,\sqrt{1+c^2\,x^2}}-$$
$$\frac{b\,c^3\,d\,x^7\,\sqrt{d+c^2\,d\,x^2}}{49\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{5\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^4\,d^2}$$

Result (type 3, 217 leaves, 7 steps):

$$\frac{2\,b\,d\,x\,\sqrt{d+c^2\,d\,x^2}}{35\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,d\,x^3\,\sqrt{d+c^2\,d\,x^2}}{105\,c\,\sqrt{1+c^2\,x^2}}-\frac{8\,b\,c\,d\,x^5\,\sqrt{d+c^2\,d\,x^2}}{175\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,d\,x^7\,\sqrt{d+c^2\,d\,x^2}}{49\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{5\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^4\,d^2}$$

Problem 136: Result optimal but 3 more steps used.

$$\int x^3\,\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 266 leaves, 4 steps):

$$\frac{2\,b\,d^2\,x\,\sqrt{d+c^2\,d\,x^2}}{63\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,d^2\,x^3\,\sqrt{d+c^2\,d\,x^2}}{189\,c\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,d^2\,x^5\,\sqrt{d+c^2\,d\,x^2}}{21\,\sqrt{1+c^2\,x^2}}-\frac{19\,b\,c^3\,d^2\,x^7\,\sqrt{d+c^2\,d\,x^2}}{441\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^5\,d^2\,x^9\,\sqrt{d+c^2\,d\,x^2}}{81\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{9/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{9\,c^4\,d^2}$$

Result (type 3, 266 leaves, 7 steps):

$$\frac{2\,b\,d^2\,x\,\sqrt{d+c^2\,d\,x^2}}{63\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,d^2\,x^3\,\sqrt{d+c^2\,d\,x^2}}{189\,c\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,d^2\,x^5\,\sqrt{d+c^2\,d\,x^2}}{21\,\sqrt{1+c^2\,x^2}}-\frac{19\,b\,c^3\,d^2\,x^7\,\sqrt{d+c^2\,d\,x^2}}{441\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^5\,d^2\,x^9\,\sqrt{d+c^2\,d\,x^2}}{81\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{9/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{9\,c^4\,d^2}$$

Problem 146: Result optimal but 1 more steps used.

$$\int \frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 192 leaves, 5 steps):

$$\frac{3\,b\,x^2\,\sqrt{1+c^2\,x^2}}{16\,c^3\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{1+c^2\,x^2}}{16\,c\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c^4\,d}+\frac{x^3\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{4\,c^2\,d}+\frac{3\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c^5\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 192 leaves, 6 steps):

$$\frac{3\,b\,x^2\,\sqrt{1+c^2\,x^2}}{16\,c^3\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{1+c^2\,x^2}}{16\,c\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c^4\,d}+\frac{x^3\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{4\,c^2\,d}+\frac{3\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c^5\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 148: Result optimal but 1 more steps used.

$$\int\frac{x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c\,\sqrt{d+c^2\,d\,x^2}}+\frac{x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^2\,d}-\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 119 leaves, 4 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c\,\sqrt{d+c^2\,d\,x^2}}+\frac{x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^2\,d}-\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 150: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 151: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x\,\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$-\frac{2\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 122 leaves, 7 steps):

$$-\frac{2\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Problem 153: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\sqrt{d+c^2 d\,x^2}}\,dx$$

Optimal (type 4, 203 leaves, 8 steps):

$$-\frac{b\,c\sqrt{1+c^2 x^2}}{2\,x\sqrt{d+c^2 d\,x^2}}-\frac{\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{2\,d\,x^2}+\frac{c^2\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\,c^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{d+c^2 d\,x^2}}-\frac{b\,c^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 203 leaves, 9 steps):

$$-\frac{b\,c\sqrt{1+c^2 x^2}}{2\,x\sqrt{d+c^2 d\,x^2}}-\frac{\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{2\,d\,x^2}+\frac{c^2\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\,c^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{d+c^2 d\,x^2}}-\frac{b\,c^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{d+c^2 d\,x^2}}$$

Problem 155: Result valid but suboptimal antiderivative.

$$\int\frac{x^5\,(a+b\,\text{ArcSinh}[c\,x])}{\left(d+c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 212 leaves, 5 steps):

$$\frac{5\,b\,x\sqrt{d+c^2 d\,x^2}}{3\,c^5 d^2\sqrt{1+c^2 x^2}}-\frac{b\,x^3\sqrt{d+c^2 d\,x^2}}{9\,c^3 d^2\sqrt{1+c^2 x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{c^6 d\sqrt{d+c^2 d\,x^2}}-\frac{2\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{c^6 d^2}+\frac{\left(d+c^2 d\,x^2\right)^{3/2}(a+b\,\text{ArcSinh}[c\,x])}{3\,c^6 d^3}+\frac{b\sqrt{d+c^2 d\,x^2}\,\text{ArcTan}[c\,x]}{c^6 d^2\sqrt{1+c^2 x^2}}$$

Result (type 3, 220 leaves, 8 steps):

$$\frac{5\,b\,x\,\sqrt{1+c^2\,x^2}}{3\,c^5\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^3\,\sqrt{1+c^2\,x^2}}{9\,c^3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{8\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^6\,d^2}+\frac{4\,x^2\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2}+\frac{b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{c^6\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 156: Result optimal but 1 more steps used.

$$\int\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 206 leaves, 7 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c^3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^4\,d^2}-\frac{3\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^5\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,c^5\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 206 leaves, 8 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c^3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^4\,d^2}-\frac{3\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^5\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,c^5\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 157: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 136 leaves, 4 steps):

$$-\frac{b\,x\,\sqrt{d+c^2\,d\,x^2}}{c^3\,d^2\,\sqrt{1+c^2\,x^2}}+\frac{a+b\,\text{ArcSinh}[c\,x]}{c^4\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^4\,d^2}-\frac{b\,\sqrt{d+c^2\,d\,x^2}\,\text{ArcTan}[c\,x]}{c^4\,d^2\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 141 leaves, 5 steps):

$$-\frac{b\,x\,\sqrt{1+c^2\,x^2}}{c^3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{2\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^4\,d^2}-\frac{b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{c^4\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 158: Result optimal but 1 more steps used.

$$\int \frac{x^2 \left(a + b\,\text{ArcSinh}[c\,x]\right)}{\left(d + c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 130 leaves, 3 steps):

$$-\frac{x\left(a + b\,\text{ArcSinh}[c\,x]\right)}{c^2\,d\,\sqrt{d + c^2\,d\,x^2}} + \frac{\sqrt{1 + c^2\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c^3\,d\,\sqrt{d + c^2\,d\,x^2}} + \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{Log}\left[1 + c^2\,x^2\right]}{2\,c^3\,d\,\sqrt{d + c^2\,d\,x^2}}$$

Result (type 3, 130 leaves, 4 steps):

$$-\frac{x\left(a + b\,\text{ArcSinh}[c\,x]\right)}{c^2\,d\,\sqrt{d + c^2\,d\,x^2}} + \frac{\sqrt{1 + c^2\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c^3\,d\,\sqrt{d + c^2\,d\,x^2}} + \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{Log}\left[1 + c^2\,x^2\right]}{2\,c^3\,d\,\sqrt{d + c^2\,d\,x^2}}$$

Problem 161: Result optimal but 1 more steps used.

$$\int \frac{a + b\,\text{ArcSinh}[c\,x]}{x\left(d + c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 194 leaves, 8 steps):

$$\frac{a + b\,\text{ArcSinh}[c\,x]}{d\,\sqrt{d + c^2\,d\,x^2}} - \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{ArcTan}[c\,x]}{d\,\sqrt{d + c^2\,d\,x^2}} - \frac{2\,\sqrt{1 + c^2\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d + c^2\,d\,x^2}} - \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d + c^2\,d\,x^2}} + \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{PolyLog}\left[2, e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d + c^2\,d\,x^2}}$$

Result (type 4, 194 leaves, 9 steps):

$$\frac{a + b\,\text{ArcSinh}[c\,x]}{d\,\sqrt{d + c^2\,d\,x^2}} - \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{ArcTan}[c\,x]}{d\,\sqrt{d + c^2\,d\,x^2}} - \frac{2\,\sqrt{1 + c^2\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d + c^2\,d\,x^2}} - \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d + c^2\,d\,x^2}} + \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{PolyLog}\left[2, e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d + c^2\,d\,x^2}}$$

Problem 162: Result valid but suboptimal antiderivative.

$$\int \frac{a + b\,\text{ArcSinh}[c\,x]}{x^2\left(d + c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 143 leaves, 5 steps):

$$-\frac{a+b\,\text{ArcSinh}[c\,x]}{d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,c^2\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{d+c^2\,d\,x^2}\,\text{Log}[x]}{d^2\,\sqrt{1+c^2\,x^2}}+\frac{b\,c\,\sqrt{d+c^2\,d\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,d^2\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 143 leaves, 7 steps):

$$-\frac{a+b\,\text{ArcSinh}[c\,x]}{d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,c^2\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{1+c^2\,x^2}\,\text{Log}[x]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 163: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\,\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 287 leaves, 11 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,d\,x^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}+$$
$$\frac{3\,c^2\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 4, 287 leaves, 12 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,d\,x^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}+$$
$$\frac{3\,c^2\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 164: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\,\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 228 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d+c^2\,d\,x^2}}{6\,d^2\,x^2\,\sqrt{1+c^2\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d\,x^3\,\sqrt{d+c^2\,d\,x^2}}+\frac{4\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,x\,\sqrt{d+c^2\,d\,x^2}}+$$
$$\frac{8\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{5\,b\,c^3\,\sqrt{d+c^2\,d\,x^2}\,\text{Log}[x]}{3\,d^2\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\sqrt{d+c^2\,d\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,d^2\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 228 leaves, 11 steps):

$$-\frac{b\,c\,\sqrt{1+c^2 x^2}}{6\,d\,x^2\,\sqrt{d+c^2 d\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d\,x^3\,\sqrt{d+c^2 d\,x^2}}+\frac{4\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,x\,\sqrt{d+c^2 d\,x^2}}+\frac{8\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{5\,b\,c^3\,\sqrt{1+c^2 x^2}\,\,\text{Log}[x]}{3\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,c^3\,\sqrt{1+c^2 x^2}\,\,\text{Log}\left[1+c^2 x^2\right]}{2\,d\,\sqrt{d+c^2 d\,x^2}}$$

Problem 165: Result optimal but 1 more steps used.

$$\int \frac{x^6\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2 d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 281 leaves, 11 steps):

$$-\frac{b}{6\,c^7\,d^2\,\sqrt{1+c^2 x^2}\,\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2 x^2}}{4\,c^5\,d^2\,\sqrt{d+c^2 d\,x^2}}-\frac{x^5\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2 d\,x^2\right)^{3/2}}-\frac{5\,x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d+c^2 d\,x^2}}+\frac{5\,x\,\sqrt{d+c^2 d\,x^2}\,\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^6\,d^3}-\frac{5\,\sqrt{1+c^2 x^2}\,\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^7\,d^2\,\sqrt{d+c^2 d\,x^2}}-\frac{7\,b\,\sqrt{1+c^2 x^2}\,\,\text{Log}\left[1+c^2 x^2\right]}{6\,c^7\,d^2\,\sqrt{d+c^2 d\,x^2}}$$

Result (type 3, 281 leaves, 12 steps):

$$-\frac{b}{6\,c^7\,d^2\,\sqrt{1+c^2 x^2}\,\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2 x^2}}{4\,c^5\,d^2\,\sqrt{d+c^2 d\,x^2}}-\frac{x^5\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2 d\,x^2\right)^{3/2}}-\frac{5\,x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d+c^2 d\,x^2}}+\frac{5\,x\,\sqrt{d+c^2 d\,x^2}\,\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^6\,d^3}-\frac{5\,\sqrt{1+c^2 x^2}\,\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^7\,d^2\,\sqrt{d+c^2 d\,x^2}}-\frac{7\,b\,\sqrt{1+c^2 x^2}\,\,\text{Log}\left[1+c^2 x^2\right]}{6\,c^7\,d^2\,\sqrt{d+c^2 d\,x^2}}$$

Problem 166: Result valid but suboptimal antiderivative.

$$\int \frac{x^5\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2 d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$\frac{b\,x\,\sqrt{d+c^2 d\,x^2}}{6\,c^5\,d^3\,\left(1+c^2 x^2\right)^{3/2}}-\frac{b\,x\,\sqrt{d+c^2 d\,x^2}}{c^5\,d^3\,\sqrt{1+c^2 x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,c^6\,d\,\left(d+c^2 d\,x^2\right)^{3/2}}+\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^6\,d^2\,\sqrt{d+c^2 d\,x^2}}+\frac{\sqrt{d+c^2 d\,x^2}\,\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^6\,d^3}-\frac{11\,b\,\sqrt{d+c^2 d\,x^2}\,\,\text{ArcTan}[c\,x]}{6\,c^6\,d^3\,\sqrt{1+c^2 x^2}}$$

Result (type 3, 225 leaves, 9 steps):

$$-\frac{b\,x^3}{6\,c^3\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{5\,b\,x\,\sqrt{1+c^2\,x^2}}{6\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-$$
$$\frac{4\,x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{8\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^6\,d^3}-\frac{11\,b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{6\,c^6\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 167: Result optimal but 1 more steps used.

$$\int\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 203 leaves, 7 steps):

$$\frac{b}{6\,c^5\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{2\,b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{3\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 203 leaves, 8 steps):

$$\frac{b}{6\,c^5\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{2\,b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{3\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 168: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 144 leaves, 4 steps):

$$-\frac{b\,x\,\sqrt{d+c^2\,d\,x^2}}{6\,c^3\,d^3\,\left(1+c^2\,x^2\right)^{3/2}}+\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,c^4\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{5\,b\,\sqrt{d+c^2\,d\,x^2}\,\text{ArcTan}[c\,x]}{6\,c^4\,d^3\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 149 leaves, 5 steps):

$$-\frac{b\,x}{6\,c^3\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{5\,b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{6\,c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 172: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x\,\left(d+c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 262 leaves, 11 steps):

$$-\frac{b c x}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{a+b \text{ArcSinh}[c x]}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{a+b \text{ArcSinh}[c x]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{7 b \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}-\frac{2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 262 leaves, 12 steps):

$$-\frac{b c x}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{a+b \text{ArcSinh}[c x]}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{a+b \text{ArcSinh}[c x]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{7 b \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}-\frac{2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Problem 173: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \text{ArcSinh}[c x]}{x^2 \left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 214 leaves, 5 steps):

$$-\frac{b c \sqrt{d+c^2 d x^2}}{6 d^3 \left(1+c^2 x^2\right)^{3/2}}-\frac{a+b \text{ArcSinh}[c x]}{d x \left(d+c^2 d x^2\right)^{3/2}}-\frac{4 c^2 x (a+b \text{ArcSinh}[c x])}{3 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{8 c^2 x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{b c \sqrt{d+c^2 d x^2} \text{Log}[x]}{d^3 \sqrt{1+c^2 x^2}}+\frac{5 b c \sqrt{d+c^2 d x^2} \text{Log}\left[1+c^2 x^2\right]}{6 d^3 \sqrt{1+c^2 x^2}}$$

Result (type 3, 214 leaves, 8 steps):

$$-\frac{b c}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{a+b \text{ArcSinh}[c x]}{d x \left(d+c^2 d x^2\right)^{3/2}}-\frac{4 c^2 x (a+b \text{ArcSinh}[c x])}{3 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{8 c^2 x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{b c \sqrt{1+c^2 x^2} \text{Log}[x]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b c \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{6 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 174: Result optimal but 1 more steps used.

$$\int \frac{a+b \text{ArcSinh}[c x]}{x^3 \left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 400 leaves, 15 steps):

$$\frac{b c}{4 d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{5 b c^3 x}{12 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{3 b c \sqrt{1+c^2 x^2}}{4 d^2 x \sqrt{d+c^2 d x^2}}-\frac{5 c^2 (a+b \text{ArcSinh}[c x])}{6 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{a+b \text{ArcSinh}[c x]}{2 d x^2 \left(d+c^2 d x^2\right)^{3/2}}-\frac{5 c^2 (a+b \text{ArcSinh}[c x])}{2 d^2 \sqrt{d+c^2 d x^2}}+\frac{13 b c^2 \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}+\frac{5 c^2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}-\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 400 leaves, 16 steps):

$$\frac{b c}{4 d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{5 b c^3 x}{12 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{3 b c \sqrt{1+c^2 x^2}}{4 d^2 x \sqrt{d+c^2 d x^2}}-\frac{5 c^2 (a+b \text{ArcSinh}[c x])}{6 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{a+b \text{ArcSinh}[c x]}{2 d x^2 \left(d+c^2 d x^2\right)^{3/2}}-\frac{5 c^2 (a+b \text{ArcSinh}[c x])}{2 d^2 \sqrt{d+c^2 d x^2}}+\frac{13 b c^2 \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}+\frac{5 c^2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}-\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \text{ArcSinh}[c x]}{x^4 \left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 297 leaves, 5 steps):

$$\frac{b c^3 \sqrt{d+c^2 d x^2}}{6 d^3 \left(1+c^2 x^2\right)^{3/2}}-\frac{b c \sqrt{d+c^2 d x^2}}{6 d^3 x^2 \sqrt{1+c^2 x^2}}-\frac{a+b \text{ArcSinh}[c x]}{3 d x^3 \left(d+c^2 d x^2\right)^{3/2}}+\frac{2 c^2 (a+b \text{ArcSinh}[c x])}{d x \left(d+c^2 d x^2\right)^{3/2}}+\frac{8 c^4 x (a+b \text{ArcSinh}[c x])}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{16 c^4 x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{8 b c^3 \sqrt{d+c^2 d x^2} \text{Log}[x]}{3 d^3 \sqrt{1+c^2 x^2}}-\frac{4 b c^3 \sqrt{d+c^2 d x^2} \text{Log}\left[1+c^2 x^2\right]}{3 d^3 \sqrt{1+c^2 x^2}}$$

Result (type 3, 297 leaves, 12 steps):

$$\frac{b c^3}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{b c \sqrt{1+c^2 x^2}}{6 d^2 x^2 \sqrt{d+c^2 d x^2}}-\frac{a+b \text{ArcSinh}[c x]}{3 d x^3 \left(d+c^2 d x^2\right)^{3/2}}+\frac{2 c^2 (a+b \text{ArcSinh}[c x])}{d x \left(d+c^2 d x^2\right)^{3/2}}+\frac{8 c^4 x (a+b \text{ArcSinh}[c x])}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{16 c^4 x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{8 b c^3 \sqrt{1+c^2 x^2} \text{Log}[x]}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{4 b c^3 \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{3 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 194: Result optimal but 1 more steps used.

$$\int \frac{x^m \left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d+c^2 d\,x^2}}\,dx$$

Optimal (type 5, 161 leaves, 1 step):

$$\frac{x^{1+m}\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2 x^2\right]}{(1+m)\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,c\,x^{2+m}\sqrt{1+c^2 x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2 x^2\right]}{\left(2+3\,m+m^2\right)\,\sqrt{d+c^2 d\,x^2}}$$

Result (type 5, 161 leaves, 2 steps):

$$\frac{x^{1+m}\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2 x^2\right]}{(1+m)\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,c\,x^{2+m}\sqrt{1+c^2 x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2 x^2\right]}{\left(2+3\,m+m^2\right)\,\sqrt{d+c^2 d\,x^2}}$$

Problem 195: Result optimal but 1 more steps used.

$$\int \frac{x^m \left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 268 leaves, 3 steps):

$$\frac{x^{1+m}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,\sqrt{d+c^2 d\,x^2}}-\frac{m\,x^{1+m}\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2 x^2\right]}{d\,(1+m)\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,c\,x^{2+m}\sqrt{1+c^2 x^2}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2 x^2\right]}{d\,(2+m)\,\sqrt{d+c^2 d\,x^2}}+\frac{b\,c\,m\,x^{2+m}\sqrt{1+c^2 x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2 x^2\right]}{d\,\left(2+3\,m+m^2\right)\,\sqrt{d+c^2 d\,x^2}}$$

Result (type 5, 268 leaves, 4 steps):

$$\frac{x^{1+m}\,(a+b\,\text{ArcSinh}[c\,x])}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{m\,x^{1+m}\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2\,x^2\right]}{d\,(1+m)\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2\,x^2\right]}{d\,(2+m)\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,m\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2\,x^2\right]}{d\,(2+3\,m+m^2)\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 196: Result optimal but 1 more steps used.

$$\int\frac{x^m\,(a+b\,\text{ArcSinh}[c\,x])}{(d+c^2\,d\,x^2)^{5/2}}\,dx$$

Optimal (type 5, 402 leaves, 5 steps):

$$\frac{x^{1+m}\,(a+b\,\text{ArcSinh}[c\,x])}{3\,d\,(d+c^2\,d\,x^2)^{3/2}}+\frac{(2-m)\,x^{1+m}\,(a+b\,\text{ArcSinh}[c\,x])}{3\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{(2-m)\,m\,x^{1+m}\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2\,x^2\right]}{3\,d^2\,(1+m)\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c\,(2-m)\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2\,x^2\right]}{3\,d^2\,(2+m)\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{Hypergeometric2F1}\left[2,\frac{2+m}{2},\frac{4+m}{2},-c^2\,x^2\right]}{3\,d^2\,(2+m)\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,(2-m)\,m\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2\,x^2\right]}{3\,d^2\,(2+3\,m+m^2)\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 5, 402 leaves, 6 steps):

$$\frac{x^{1+m}\,(a+b\,\text{ArcSinh}[c\,x])}{3\,d\,(d+c^2\,d\,x^2)^{3/2}}+\frac{(2-m)\,x^{1+m}\,(a+b\,\text{ArcSinh}[c\,x])}{3\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{(2-m)\,m\,x^{1+m}\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2\,x^2\right]}{3\,d^2\,(1+m)\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c\,(2-m)\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2\,x^2\right]}{3\,d^2\,(2+m)\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{Hypergeometric2F1}\left[2,\frac{2+m}{2},\frac{4+m}{2},-c^2\,x^2\right]}{3\,d^2\,(2+m)\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,(2-m)\,m\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2\,x^2\right]}{3\,d^2\,(2+3\,m+m^2)\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 252: Result valid but suboptimal antiderivative.

$$\int \left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 \, dx$$

Optimal (type 3, 300 leaves, 16 steps):

$$\frac{245 b^2 \pi^{5/2} x \sqrt{1 + c^2 x^2}}{1152} + \frac{65 b^2 \pi^{5/2} x \left(1 + c^2 x^2\right)^{3/2}}{1728} + \frac{1}{108} b^2 \pi^{5/2} x \left(1 + c^2 x^2\right)^{5/2} - \frac{115 b^2 \pi^{5/2} \operatorname{ArcSinh}[c x]}{1152 c} - \frac{5}{16} b c \pi^{5/2} x^2 \left(a + b \operatorname{ArcSinh}[c x]\right) - \frac{5 b \pi^{5/2} \left(1 + c^2 x^2\right)^2 \left(a + b \operatorname{ArcSinh}[c x]\right)}{48 c} - \frac{b \pi^{5/2} \left(1 + c^2 x^2\right)^3 \left(a + b \operatorname{ArcSinh}[c x]\right)}{18 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{5}{24} \pi x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{1}{6} x \left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{5 \pi^{5/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^3}{48 b c}$$

Result (type 3, 420 leaves, 16 steps):

$$\frac{245 b^2 \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{1152} + \frac{65 b^2 \pi^2 x \left(1 + c^2 x^2\right) \sqrt{\pi + c^2 \pi x^2}}{1728} + \frac{1}{108} b^2 \pi^2 x \left(1 + c^2 x^2\right)^2 \sqrt{\pi + c^2 \pi x^2} - \frac{115 b^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{ArcSinh}[c x]}{1152 c \sqrt{1 + c^2 x^2}} - \frac{5 b c \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{16 \sqrt{1 + c^2 x^2}} - \frac{5 b \pi^2 \left(1 + c^2 x^2\right)^{3/2} \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{48 c} - \frac{b \pi^2 \left(1 + c^2 x^2\right)^{5/2} \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)}{18 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{5}{24} \pi x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{1}{6} x \left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{5 \pi^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)^3}{48 b c \sqrt{1 + c^2 x^2}}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 \, dx$$

Optimal (type 3, 210 leaves, 10 steps):

$$\frac{15}{64} b^2 \pi^{3/2} x \sqrt{1 + c^2 x^2} + \frac{1}{32} b^2 \pi^{3/2} x \left(1 + c^2 x^2\right)^{3/2} - \frac{9 b^2 \pi^{3/2} \operatorname{ArcSinh}[c x]}{64 c} - \frac{3}{8} b c \pi^{3/2} x^2 \left(a + b \operatorname{ArcSinh}[c x]\right) - \frac{b \pi^{3/2} \left(1 + c^2 x^2\right)^2 \left(a + b \operatorname{ArcSinh}[c x]\right)}{8 c} + \frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{1}{4} x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^2 + \frac{\pi^{3/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^3}{8 b c}$$

Result (type 3, 294 leaves, 10 steps):

$$\frac{15}{64} b^2 \pi x \sqrt{\pi + c^2 \pi x^2} + \frac{1}{32} b^2 \pi x \left(1 + c^2 x^2\right) \sqrt{\pi + c^2 \pi x^2} - \frac{9 b^2 \pi \sqrt{\pi + c^2 \pi x^2} \text{ArcSinh}[c x]}{64 c \sqrt{1 + c^2 x^2}} -$$
$$\frac{3 b c \pi x^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)}{8 \sqrt{1 + c^2 x^2}} - \frac{b \pi \left(1 + c^2 x^2\right)^{3/2} \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)}{8 c} +$$
$$\frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)^2 + \frac{1}{4} x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \text{ArcSinh}[c x]\right)^2 + \frac{\pi \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)^3}{8 b c \sqrt{1 + c^2 x^2}}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)^2 dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{1}{4} b^2 \sqrt{\pi} x \sqrt{1 + c^2 x^2} - \frac{b^2 \sqrt{\pi} \text{ArcSinh}[c x]}{4 c} - \frac{1}{2} b c \sqrt{\pi} x^2 \left(a + b \text{ArcSinh}[c x]\right) + \frac{1}{2} x \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)^2 + \frac{\sqrt{\pi} \left(a + b \text{ArcSinh}[c x]\right)^3}{6 b c}$$

Result (type 3, 184 leaves, 5 steps):

$$\frac{1}{4} b^2 x \sqrt{\pi + c^2 \pi x^2} - \frac{b^2 \sqrt{\pi + c^2 \pi x^2} \text{ArcSinh}[c x]}{4 c \sqrt{1 + c^2 x^2}} - \frac{b c x^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)}{2 \sqrt{1 + c^2 x^2}} +$$
$$\frac{1}{2} x \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)^2 + \frac{\sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)^3}{6 b c \sqrt{1 + c^2 x^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \text{ArcSinh}[c x]\right)^2}{\left(\pi + c^2 \pi x^2\right)^{3/2}} dx$$

Optimal (type 4, 104 leaves, 6 steps):

$$\frac{\left(a + b \text{ArcSinh}[c x]\right)^2}{c \pi^{3/2}} + \frac{x \left(a + b \text{ArcSinh}[c x]\right)^2}{\pi \sqrt{\pi + c^2 \pi x^2}} - \frac{2 b \left(a + b \text{ArcSinh}[c x]\right) \text{Log}\left[1 + e^{2 \text{ArcSinh}[c x]}\right]}{c \pi^{3/2}} - \frac{b^2 \text{PolyLog}\left[2, -e^{2 \text{ArcSinh}[c x]}\right]}{c \pi^{3/2}}$$

Result (type 4, 179 leaves, 6 steps):

$$\frac{x\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c\,\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2\,b\sqrt{1+c^2x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{c\,\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{c\,\pi\sqrt{\pi+c^2\pi x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 204 leaves, 9 steps):

$$-\frac{b^2x}{3\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{b\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c\,\pi^{5/2}\left(1+c^2x^2\right)}+\frac{2\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\,c\,\pi^{5/2}}+\frac{x\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{4\,b\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{3\,c\,\pi^{5/2}}-\frac{2\,b^2\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{3\,c\,\pi^{5/2}}$$

Result (type 4, 292 leaves, 9 steps):

$$-\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c\,\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{2\,x\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{2\sqrt{1+c^2x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\,c\,\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{4\,b\sqrt{1+c^2x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{3\,c\,\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{2\,b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{3\,c\,\pi^2\sqrt{\pi+c^2\pi x^2}}$$

Problem 291: Result optimal but 1 more steps used.

$$\int\frac{x^4\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\sqrt{d+c^2d\,x^2}}\,dx$$

Optimal (type 3, 323 leaves, 10 steps):

$$-\frac{15\,b^2x\left(1+c^2x^2\right)}{64\,c^4\sqrt{d+c^2d\,x^2}}+\frac{b^2x^3\left(1+c^2x^2\right)}{32\,c^2\sqrt{d+c^2d\,x^2}}+\frac{15\,b^2\sqrt{1+c^2x^2}\,\text{ArcSinh}[c\,x]}{64\,c^5\sqrt{d+c^2d\,x^2}}+\frac{3\,b\,x^2\sqrt{1+c^2x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c^3\sqrt{d+c^2d\,x^2}}-\frac{b\,x^4\sqrt{1+c^2x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c\sqrt{d+c^2d\,x^2}}-\frac{3\,x\sqrt{d+c^2d\,x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{8\,c^4\,d}+\frac{x^3\sqrt{d+c^2d\,x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,c^2\,d}+\frac{\sqrt{1+c^2x^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^3}{8\,b\,c^5\sqrt{d+c^2d\,x^2}}$$

Result (type 3, 323 leaves, 11 steps):

$$-\frac{15 b^2 x \left(1+c^2 x^2\right)}{64 c^4 \sqrt{d+c^2 d x^2}}+\frac{b^2 x^3 \left(1+c^2 x^2\right)}{32 c^2 \sqrt{d+c^2 d x^2}}+\frac{15 b^2 \sqrt{1+c^2 x^2}\, \text{ArcSinh}[c x]}{64 c^5 \sqrt{d+c^2 d x^2}}+\frac{3 b x^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)}{8 c^3 \sqrt{d+c^2 d x^2}}-\frac{b x^4 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)}{8 c \sqrt{d+c^2 d x^2}}-\frac{3 x \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{8 c^4 d}+\frac{x^3 \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{4 c^2 d}+\frac{\sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^3}{8 b c^5 \sqrt{d+c^2 d x^2}}$$

Problem 293: Result optimal but 1 more steps used.

$$\int \frac{x^2 \left(a+b \text{ArcSinh}[c x]\right)^2}{\sqrt{d+c^2 d x^2}} \, dx$$

Optimal (type 3, 204 leaves, 5 steps):

$$\frac{b^2 x \left(1+c^2 x^2\right)}{4 c^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 \sqrt{1+c^2 x^2}\, \text{ArcSinh}[c x]}{4 c^3 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)}{2 c \sqrt{d+c^2 d x^2}}+\frac{x \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{2 c^2 d}-\frac{\sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^3}{6 b c^3 \sqrt{d+c^2 d x^2}}$$

Result (type 3, 204 leaves, 6 steps):

$$\frac{b^2 x \left(1+c^2 x^2\right)}{4 c^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 \sqrt{1+c^2 x^2}\, \text{ArcSinh}[c x]}{4 c^3 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)}{2 c \sqrt{d+c^2 d x^2}}+\frac{x \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{2 c^2 d}-\frac{\sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^3}{6 b c^3 \sqrt{d+c^2 d x^2}}$$

Problem 295: Result optimal but 1 more steps used.

$$\int \frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{\sqrt{d+c^2 d x^2}} \, dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^3}{3 b c \sqrt{d+c^2 d x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^3}{3\,b\,c\,\sqrt{d+c^2 d\,x^2}}$$

Problem 296: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcSinh}[c\,x])^2}{x\,\sqrt{d+c^2 d\,x^2}}\,dx$$

Optimal (type 4, 223 leaves, 8 steps):

$$-\frac{2\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^2\ \text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{2\,b\,\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])\ \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{2\,b\,\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])\ \text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{2\,b^2\sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{2\,b^2\sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 223 leaves, 9 steps):

$$-\frac{2\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^2\ \text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{2\,b\,\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])\ \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{2\,b\,\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])\ \text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{2\,b^2\sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{2\,b^2\sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Problem 298: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcSinh}[c\,x])^2}{x^3\,\sqrt{d+c^2 d\,x^2}}\,dx$$

Optimal (type 4, 360 leaves, 13 steps):

$$-\frac{b\,c\,\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])}{x\,\sqrt{d+c^2 d\,x^2}}-\frac{\sqrt{d+c^2 d\,x^2}\ (a+b\,\text{ArcSinh}[c\,x])^2}{2\,d\,x^2}+\frac{c^2\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^2\ \text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2 c^2\sqrt{1+c^2 x^2}\ \text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\,c^2\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])\ \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b\,c^2\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])\ \text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2 c^2\sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b^2 c^2\sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 360 leaves, 14 steps):

$$-\frac{b\,c\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x\,\sqrt{d+c^2 d\,x^2}}-\frac{\sqrt{d+c^2 d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,x^2}+\frac{c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b\,c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Problem 301: Result optimal but 1 more steps used.

$$\int\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\left(d+c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 400 leaves, 14 steps):

$$\frac{b^2\,x\,\left(1+c^2 x^2\right)}{4\,c^4\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,\sqrt{1+c^2 x^2}\,\text{ArcSinh}[c\,x]}{4\,c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^3\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c^2\,d\,\sqrt{d+c^2 d\,x^2}}+\frac{\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}+\frac{3\,x\,\sqrt{d+c^2 d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,c^4\,d^2}-\frac{\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^3}{2\,b\,c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{2\,b\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 400 leaves, 15 steps):

$$\frac{b^2\,x\,\left(1+c^2 x^2\right)}{4\,c^4\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,\sqrt{1+c^2 x^2}\,\text{ArcSinh}[c\,x]}{4\,c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^3\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c^2\,d\,\sqrt{d+c^2 d\,x^2}}+\frac{\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}+\frac{3\,x\,\sqrt{d+c^2 d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,c^4\,d^2}-\frac{\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^3}{2\,b\,c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{2\,b\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}$$

Problem 303: Result optimal but 1 more steps used.

$$\int\frac{x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\left(d+c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 233 leaves, 7 steps):

$$-\frac{x\,(a+b\,\text{ArcSinh}[c\,x])^2}{c^2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^2}{c^3\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^3}{3\,b\,c^3\,d\,\sqrt{d+c^2\,d\,x^2}}+$$
$$\frac{2\,b\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^3\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,\,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^3\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 4, 233 leaves, 8 steps):

$$-\frac{x\,(a+b\,\text{ArcSinh}[c\,x])^2}{c^2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^2}{c^3\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^3}{3\,b\,c^3\,d\,\sqrt{d+c^2\,d\,x^2}}+$$
$$\frac{2\,b\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^3\,d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,\,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^3\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 306: Result optimal but 1 more steps used.

$$\int\frac{(a+b\,\text{ArcSinh}[c\,x])^2}{x\,\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 412 leaves, 15 steps):

$$\frac{(a+b\,\text{ArcSinh}[c\,x])^2}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{4\,b\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}-$$
$$\frac{2\,b\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{2\,i\,b^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}-$$
$$\frac{2\,i\,b^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{2\,b\,\sqrt{1+c^2\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+$$
$$\frac{2\,b^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[3,\,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,b^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[3,\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 4, 412 leaves, 16 steps):

$$\frac{(a+b\,\text{ArcSinh}[c\,x])^2}{d\sqrt{d+c^2 d x^2}}-\frac{4\,b\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-\frac{2\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-$$
$$\frac{2\,b\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}+\frac{2\,i\,b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-$$
$$\frac{2\,i\,b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}+\frac{2\,b\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}+$$
$$\frac{2\,b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-\frac{2\,b^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}$$

Problem 308: Result optimal but 1 more steps used.

$$\int\frac{(a+b\,\text{ArcSinh}[c\,x])^2}{x^3\,(d+c^2 d x^2)^{3/2}}\,dx$$

Optimal (type 4, 573 leaves, 26 steps):

$$-\frac{b\,c\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])}{d\,x\sqrt{d+c^2 d x^2}}-\frac{3\,c^2\,(a+b\,\text{ArcSinh}[c\,x])^2}{2\,d\sqrt{d+c^2 d x^2}}-$$
$$\frac{(a+b\,\text{ArcSinh}[c\,x])^2}{2\,d\,x^2\sqrt{d+c^2 d x^2}}+\frac{4\,b\,c^2\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}+$$
$$\frac{3\,c^2\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-\frac{b^2\,c^2\sqrt{1+c^2x^2}\,\text{ArcTanh}\left[\sqrt{1+c^2x^2}\right]}{d\sqrt{d+c^2 d x^2}}+$$
$$\frac{3\,b\,c^2\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-\frac{2\,i\,b^2\,c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}+$$
$$\frac{2\,i\,b^2\,c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-\frac{3\,b\,c^2\sqrt{1+c^2x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}-$$
$$\frac{3\,b^2\,c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}+\frac{3\,b^2\,c^2\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d x^2}}$$

Result (type 4, 573 leaves, 27 steps):

$$-\frac{b\,c\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,x\,\sqrt{d+c^2d\,x^2}}-\frac{3\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,\sqrt{d+c^2d\,x^2}}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,x^2\,\sqrt{d+c^2d\,x^2}}+\frac{4\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{ArcTanh}\left[\sqrt{1+c^2x^2}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{2\,i\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{2\,i\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{3\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{3\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}$$

Problem 311: Result optimal but 1 more steps used.

$$\int\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\left(d+c^2d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 398 leaves, 16 steps):

$$-\frac{b^2\,x}{3\,c^4\,d^2\,\sqrt{d+c^2d\,x^2}}+\frac{b^2\,\sqrt{1+c^2x^2}\,\text{ArcSinh}[c\,x]}{3\,c^5\,d^2\,\sqrt{d+c^2d\,x^2}}-\frac{b\,x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^3\,d^2\,\sqrt{1+c^2x^2}\,\sqrt{d+c^2d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\,c^2\,d\,\left(d+c^2d\,x^2\right)^{3/2}}-\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c^4\,d^2\,\sqrt{d+c^2d\,x^2}}-\frac{4\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{3\,c^5\,d^2\,\sqrt{d+c^2d\,x^2}}+\frac{\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^3}{3\,b\,c^5\,d^2\,\sqrt{d+c^2d\,x^2}}+\frac{8\,b\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{3\,c^5\,d^2\,\sqrt{d+c^2d\,x^2}}+\frac{4\,b^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{3\,c^5\,d^2\,\sqrt{d+c^2d\,x^2}}$$

Result (type 4, 398 leaves, 17 steps):

$$-\frac{b^2 x}{3 c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{b^2 \sqrt{1+c^2 x^2}\ \text{ArcSinh}[c x]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \left(a+b\,\text{ArcSinh}[c x]\right)}{3 c^3 d^2 \sqrt{1+c^2 x^2}\ \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b\,\text{ArcSinh}[c x]\right)^2}{3 c^2 d \left(d+c^2 d x^2\right)^{3/2}}-$$
$$\frac{x \left(a+b\,\text{ArcSinh}[c x]\right)^2}{c^4 d^2 \sqrt{d+c^2 d x^2}}-\frac{4 \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^2}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^3}{3 b c^5 d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{8 b \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2\,\text{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{4 b^2 \sqrt{1+c^2 x^2}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 316: Result optimal but 1 more steps used.

$$\int \frac{\left(a+b\,\text{ArcSinh}[c x]\right)^2}{x \left(d+c^2 d x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 518 leaves, 24 steps):

$$-\frac{b^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c x \left(a+b\,\text{ArcSinh}[c x]\right)}{3 d^2 \sqrt{1+c^2 x^2}\ \sqrt{d+c^2 d x^2}}+\frac{\left(a+b\,\text{ArcSinh}[c x]\right)^2}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{\left(a+b\,\text{ArcSinh}[c x]\right)^2}{d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{14 b \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{2 \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{2 b \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{7 i b^2 \sqrt{1+c^2 x^2}\ \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{7 i b^2 \sqrt{1+c^2 x^2}\ \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{2 b \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{2 b^2 \sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{2 b^2 \sqrt{1+c^2 x^2}\ \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 518 leaves, 25 steps):

$$-\frac{b^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{(a+b \text{ArcSinh}[c x])^2}{3 d (d+c^2 d x^2)^{3/2}}+\frac{(a+b \text{ArcSinh}[c x])^2}{d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{14 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{2 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{7 i b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{7 i b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{2 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{2 b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{2 b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Problem 318: Result optimal but 1 more steps used.

$$\int \frac{(a+b \text{ArcSinh}[c x])^2}{x^3 (d+c^2 d x^2)^{5/2}} \, dx$$

Optimal (type 4, 687 leaves, 38 steps):

$$\frac{b^2 c^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c (a+b \text{ArcSinh}[c x])}{d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{2 b c^3 x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{5 c^2 (a+b \text{ArcSinh}[c x])^2}{6 d (d+c^2 d x^2)^{3/2}}-$$
$$\frac{(a+b \text{ArcSinh}[c x])^2}{2 d x^2 (d+c^2 d x^2)^{3/2}}-\frac{5 c^2 (a+b \text{ArcSinh}[c x])^2}{2 d^2 \sqrt{d+c^2 d x^2}}+\frac{26 b c^2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{5 c^2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 c^2 \sqrt{1+c^2 x^2} \text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{5 b c^2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{5 b c^2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 687 leaves, 39 steps):

$$\frac{b^2 c^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c \left(a+b \text{ArcSinh}[c x]\right)}{d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{2 b c^3 x \left(a+b \text{ArcSinh}[c x]\right)}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)^2}{6 d \left(d+c^2 d x^2\right)^{3/2}}-$$

$$\frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{2 d x^2 \left(d+c^2 d x^2\right)^{3/2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)^2}{2 d^2 \sqrt{d+c^2 d x^2}}+\frac{26 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+$$

$$\frac{5 c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 c^2 \sqrt{1+c^2 x^2} \text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$

$$\frac{5 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+$$

$$\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{5 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-$$

$$\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} \left(a+b \text{ArcSinh}[c x]\right)^2 dx$$

Optimal (type 8, 935 leaves, 12 steps) :

$$\frac{10\, b^2 c^2 d^2 x^{3+m} \sqrt{d+c^2 d x^2}}{(4+m)^3 (6+m)} + \frac{2\, b^2 c^2 d^2 \left(52+15\, m+m^2\right) x^{3+m} \sqrt{d+c^2 d x^2}}{(4+m)^2 (6+m)^3} + \frac{2\, b^2 c^4 d^2 x^{5+m} \sqrt{d+c^2 d x^2}}{(6+m)^3} -$$

$$\frac{30\, b\, c\, d^2 x^{2+m} \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c\, x]\right)}{(2+m)^2 (4+m) (6+m) \sqrt{1+c^2 x^2}} - \frac{10\, b\, c\, d^2 x^{2+m} \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c\, x]\right)}{(6+m) \left(8+6\, m+m^2\right) \sqrt{1+c^2 x^2}} - \frac{2\, b\, c\, d^2 x^{2+m} \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c\, x]\right)}{\left(12+8\, m+m^2\right) \sqrt{1+c^2 x^2}} -$$

$$\frac{10\, b\, c^3 d^2 x^{4+m} \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c\, x]\right)}{(4+m)^2 (6+m) \sqrt{1+c^2 x^2}} - \frac{4\, b\, c^3 d^2 x^{4+m} \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c\, x]\right)}{(4+m) (6+m) \sqrt{1+c^2 x^2}} -$$

$$\frac{2\, b\, c^5 d^2 x^{6+m} \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c\, x]\right)}{(6+m)^2 \sqrt{1+c^2 x^2}} + \frac{15\, d^2 x^{1+m} \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c\, x]\right)^2}{(6+m) \left(8+6\, m+m^2\right)} + \frac{5\, d\, x^{1+m} \left(d+c^2 d x^2\right)^{3/2} \left(a+b\, \text{ArcSinh}[c\, x]\right)^2}{(4+m) (6+m)} +$$

$$\frac{x^{1+m} \left(d+c^2 d x^2\right)^{5/2} \left(a+b\, \text{ArcSinh}[c\, x]\right)^2}{6+m} + \frac{30\, b^2 c^2 d^2 x^{3+m} \sqrt{d+c^2 d x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) (4+m) (6+m) \sqrt{1+c^2 x^2}} +$$

$$\frac{10\, b^2 c^2 d^2 (10+3\, m)\, x^{3+m} \sqrt{d+c^2 d x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m) (3+m) (4+m)^3 (6+m) \sqrt{1+c^2 x^2}} +$$

$$\frac{2\, b^2 c^2 d^2 \left(264+130\, m+15\, m^2\right) x^{3+m} \sqrt{d+c^2 d x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m) (3+m) (4+m)^2 (6+m)^3 \sqrt{1+c^2 x^2}} + \frac{15\, d^3\, \text{Unintegrable}\left[\frac{x^m \left(a+b\, \text{ArcSinh}[c\, x]\right)^2}{\sqrt{d+c^2 d x^2}}, x\right]}{(6+m) \left(8+6\, m+m^2\right)}$$

Result (type 8, 30 leaves, 0 steps) :

$$\text{Unintegrable}\left[x^m \left(d+c^2 d x^2\right)^{5/2} \left(a+b\, \text{ArcSinh}[c\, x]\right)^2, x\right]$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int x^m \left(d+c^2 d x^2\right)^{3/2} \left(a+b\, \text{ArcSinh}[c\, x]\right)^2 dx$$

Optimal (type 8, 487 leaves, 7 steps) :

$$\frac{2\, b^2 c^2 d\, x^{3+m} \sqrt{d+c^2 d\, x^2}}{(4+m)^3} - \frac{6\, b\, c\, d\, x^{2+m} \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])}{(2+m)^2 (4+m) \sqrt{1+c^2 x^2}} - \frac{2\, b\, c\, d\, x^{2+m} \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])}{(8+6\, m+m^2) \sqrt{1+c^2 x^2}} - \frac{2\, b\, c^3 d\, x^{4+m} \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])}{(4+m)^2 \sqrt{1+c^2 x^2}} + \frac{3\, d\, x^{1+m} \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])^2}{8+6\, m+m^2} + \frac{x^{1+m} (d+c^2 d\, x^2)^{3/2} (a+b\, \text{ArcSinh}[c\, x])^2}{4+m} + \frac{6\, b^2 c^2 d\, x^{3+m} \sqrt{d+c^2 d\, x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) (4+m) \sqrt{1+c^2 x^2}} + \frac{2\, b^2 c^2 d\, (10+3\, m)\, x^{3+m} \sqrt{d+c^2 d\, x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m) (3+m) (4+m)^3 \sqrt{1+c^2 x^2}} + \frac{3\, d^2\, \text{Unintegrable}\left[\frac{x^m (a+b\, \text{ArcSinh}[c\, x])^2}{\sqrt{d+c^2 d\, x^2}}, x\right]}{8+6\, m+m^2}$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[x^m (d+c^2 d\, x^2)^{3/2} (a+b\, \text{ArcSinh}[c\, x])^2, x\right]$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int x^m \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])^2\, dx$$

Optimal (type 8, 198 leaves, 3 steps):

$$-\frac{2\, b\, c\, x^{2+m} \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])}{(2+m)^2 \sqrt{1+c^2 x^2}} + \frac{x^{1+m} \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])^2}{2+m} + \frac{2\, b^2 c^2 x^{3+m} \sqrt{d+c^2 d\, x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) \sqrt{1+c^2 x^2}} + \frac{d\, \text{Unintegrable}\left[\frac{x^m (a+b\, \text{ArcSinh}[c\, x])^2}{\sqrt{d+c^2 d\, x^2}}, x\right]}{2+m}$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[x^m \sqrt{d+c^2 d\, x^2}\, (a+b\, \text{ArcSinh}[c\, x])^2, x\right]$$

Problem 337: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSinh}[a\, x]^3}{\sqrt{c+a^2 c\, x^2}}\, dx$$

Optimal (type 3, 40 leaves, 1 step):

$$\frac{\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^4}{4\,a\,\sqrt{c+a^2\,c\,x^2}}$$

Result (type 3, 40 leaves, 2 steps):

$$\frac{\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^4}{4\,a\,\sqrt{c+a^2\,c\,x^2}}$$

Problem 413: Result valid but suboptimal antiderivative.

$$\int \frac{x\,\sqrt{1+c^2\,x^2}}{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 149 leaves, 14 steps):

$$-\frac{x\,\left(1+c^2\,x^2\right)}{b\,c\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}+\frac{\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{4\,b^2\,c^2}+\frac{3\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{4\,b^2\,c^2}-\frac{\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{4\,b^2\,c^2}-\frac{3\,\text{Sinh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{4\,b^2\,c^2}$$

Result (type 4, 198 leaves, 14 steps):

$$-\frac{x\,\left(1+c^2\,x^2\right)}{b\,c\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}-\frac{3\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c\,x]\right]}{4\,b^2\,c^2}+\frac{3\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcSinh}[c\,x]\right]}{4\,b^2\,c^2}+\frac{\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{b^2\,c^2}+\frac{3\,\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c\,x]\right]}{4\,b^2\,c^2}-\frac{3\,\text{Sinh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcSinh}[c\,x]\right]}{4\,b^2\,c^2}-\frac{\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{b^2\,c^2}$$

Problem 474: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcSinh}[a\,x]}}{\sqrt{c+a^2\,c\,x^2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2\,\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{3/2}}{3\,a\,\sqrt{c+a^2\,c\,x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 \sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{3/2}}{3\,a \sqrt{c+a^2 c\,x^2}}$$

Problem 479: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSinh}[a\,x]^{3/2}}{\sqrt{c+a^2 c\,x^2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 \sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{5/2}}{5\,a \sqrt{c+a^2 c\,x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 \sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{5/2}}{5\,a \sqrt{c+a^2 c\,x^2}}$$

Problem 483: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSinh}[a\,x]^{5/2}}{\sqrt{c+a^2 c\,x^2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 \sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{7/2}}{7\,a \sqrt{c+a^2 c\,x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 \sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{7/2}}{7\,a \sqrt{c+a^2 c\,x^2}}$$

Problem 487: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcSinh}\left[\frac{x}{a}\right]}}{\sqrt{a^2+x^2}}\,dx$$

Optimal (type 3, 39 leaves, 1 step):

$$\frac{2 a \sqrt{1+\frac{x^2}{a^2}}\ \text{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2+x^2}}$$

Result (type 3, 39 leaves, 2 steps) :

$$\frac{2 a \sqrt{1+\frac{x^2}{a^2}}\ \text{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2+x^2}}$$

Problem 492: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2+x^2}}\, dx$$

Optimal (type 3, 39 leaves, 1 step) :

$$\frac{2 a \sqrt{1+\frac{x^2}{a^2}}\ \text{ArcSinh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2+x^2}}$$

Result (type 3, 39 leaves, 2 steps) :

$$\frac{2 a \sqrt{1+\frac{x^2}{a^2}}\ \text{ArcSinh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2+x^2}}$$

Problem 495: Result optimal but 1 more steps used.

$$\int \frac{\left(c+a^2 c x^2\right)^{5/2}}{\sqrt{\text{ArcSinh}[a x]}}\, dx$$

Optimal (type 4, 396 leaves, 18 steps) :

$$\frac{5\,c^2\sqrt{c+a^2 c x^2}\sqrt{\text{ArcSinh}[a\,x]}}{8\,a\sqrt{1+a^2 x^2}}+\frac{3\,c^2\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{15\,c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{6}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{3\,c^2\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{15\,c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{6}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}$$

Result (type 4, 396 leaves, 19 steps):

$$\frac{5\,c^2\sqrt{c+a^2 c x^2}\sqrt{\text{ArcSinh}[a\,x]}}{8\,a\sqrt{1+a^2 x^2}}+\frac{3\,c^2\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{15\,c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{6}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{3\,c^2\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{15\,c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}+\frac{c^2\sqrt{\frac{\pi}{6}}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{6}\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\sqrt{1+a^2 x^2}}$$

Problem 496: Result optimal but 1 more steps used.

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{\sqrt{\text{ArcSinh}[a\,x]}}\,dx$$

Optimal (type 4, 264 leaves, 13 steps):

$$\frac{3\,c\sqrt{c+a^2 c x^2}\sqrt{\text{ArcSinh}[a\,x]}}{4\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\sqrt{1+a^2 x^2}}$$

Result (type 4, 264 leaves, 14 steps):

$$\frac{3\,c\sqrt{c+a^2 c x^2}\sqrt{\text{ArcSinh}[a\,x]}}{4\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\pi}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[2\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\sqrt{1+a^2 x^2}}+\frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\sqrt{1+a^2 x^2}}$$

Problem 497: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{c+a^2 c x^2}}{\sqrt{\text{ArcSinh}[a x]}} \, dx$$

Optimal (type 4, 156 leaves, 8 steps) :

$$\frac{\sqrt{c+a^2 c x^2}\ \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{1+a^2 x^2}}+\frac{\sqrt{\frac{\pi}{2}}\ \sqrt{c+a^2 c x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcSinh}[a x]}\ \right]}{4 a \sqrt{1+a^2 x^2}}+\frac{\sqrt{\frac{\pi}{2}}\ \sqrt{c+a^2 c x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcSinh}[a x]}\ \right]}{4 a \sqrt{1+a^2 x^2}}$$

Result (type 4, 156 leaves, 9 steps) :

$$\frac{\sqrt{c+a^2 c x^2}\ \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{1+a^2 x^2}}+\frac{\sqrt{\frac{\pi}{2}}\ \sqrt{c+a^2 c x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcSinh}[a x]}\ \right]}{4 a \sqrt{1+a^2 x^2}}+\frac{\sqrt{\frac{\pi}{2}}\ \sqrt{c+a^2 c x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcSinh}[a x]}\ \right]}{4 a \sqrt{1+a^2 x^2}}$$

Problem 498: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c+a^2 c x^2}\ \sqrt{\text{ArcSinh}[a x]}} \, dx$$

Optimal (type 3, 40 leaves, 1 step) :

$$\frac{2 \sqrt{1+a^2 x^2}\ \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{c+a^2 c x^2}}$$

Result (type 3, 40 leaves, 2 steps) :

$$\frac{2 \sqrt{1+a^2 x^2}\ \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{c+a^2 c x^2}}$$

Problem 504: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c+a^2 c x^2}\ \text{ArcSinh}[a x]^{3/2}} \, dx$$

Optimal (type 3, 40 leaves, 1 step) :

$$-\frac{2 \sqrt{1+a^2 x^2}}{a \sqrt{c+a^2 c x^2}\ \sqrt{\text{ArcSinh}[a x]}}$$

Result (type 3, 40 leaves, 2 steps) :

$$-\frac{2\sqrt{1+a^2 x^2}}{a\sqrt{c+a^2 c x^2}\sqrt{\text{ArcSinh}[a x]}}$$

Problem 509: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c+a^2 c x^2}\,\text{ArcSinh}[a x]^{5/2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$-\frac{2\sqrt{1+a^2 x^2}}{3 a\sqrt{c+a^2 c x^2}\,\text{ArcSinh}[a x]^{3/2}}$$

Result (type 3, 42 leaves, 2 steps):

$$-\frac{2\sqrt{1+a^2 x^2}}{3 a\sqrt{c+a^2 c x^2}\,\text{ArcSinh}[a x]^{3/2}}$$

Problem 512: Result optimal but 1 more steps used.

$$\int x^2\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\,dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$-\frac{\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^{1+n}}{8 b c^3 (1+n)\sqrt{1+c^2 x^2}}+\frac{2^{-2(3+n)}\,e^{-\frac{4a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$
$$\frac{2^{-2(3+n)}\,e^{\frac{4a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}$$

Result (type 4, 235 leaves, 7 steps):

$$-\frac{\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^{1+n}}{8 b c^3 (1+n)\sqrt{1+c^2 x^2}}+\frac{2^{-2(3+n)}\,e^{-\frac{4a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$
$$\frac{2^{-2(3+n)}\,e^{\frac{4a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}$$

Problem 513: Result optimal but 1 more steps used.

$$\int x \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \, dx$$

Optimal (type 4, 355 leaves, 9 steps):

$$\frac{3^{-1-n} e^{-\frac{3a}{b}} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}+$$
$$\frac{e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}+$$
$$\frac{e^{a/b} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}+$$
$$\frac{3^{-1-n} e^{\frac{3a}{b}} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}$$

Result (type 4, 355 leaves, 10 steps):

$$\frac{3^{-1-n} e^{-\frac{3a}{b}} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}+$$
$$\frac{e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}+$$
$$\frac{e^{a/b} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}+$$
$$\frac{3^{-1-n} e^{\frac{3a}{b}} \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1+c^2 x^2}}$$

Problem 514: Result optimal but 1 more steps used.

$$\int \sqrt{d+c^2 d x^2} \, (a+b \operatorname{ArcSinh}[c x])^n \, dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$\frac{\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{1+c^2 x^2}}+\frac{2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c\,\sqrt{1+c^2 x^2}}-\frac{2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c\,\sqrt{1+c^2 x^2}}$$

Result (type 4, 235 leaves, 7 steps):

$$\frac{\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{1+c^2 x^2}}+\frac{2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c\,\sqrt{1+c^2 x^2}}-\frac{2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c\,\sqrt{1+c^2 x^2}}$$

Problem 515: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n}{x}\,dx$$

Optimal (type 8, 198 leaves, 6 steps):

$$\frac{d\,e^{-\frac{a}{b}}\,\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{2\,\sqrt{d+c^2 d x^2}}+\frac{d\,e^{a/b}\,\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{2\,\sqrt{d+c^2 d x^2}}+d\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcSinh}[c\,x])^n}{x\,\sqrt{d+c^2 d x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n}{x},x\right]$$

Problem 516: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n}{x^2}\,dx$$

Optimal (type 8, 83 leaves, 3 steps):

$$\frac{c\,d\,\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])^{1+n}}{b\,(1+n)\,\sqrt{d+c^2 d\,x^2}}+d\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcSinh}[c\,x])^n}{x^2\sqrt{d+c^2 d\,x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n}{x^2},x\right]$$

Problem 517: Result optimal but 1 more steps used.

$$\int x^2\left(d+c^2 d\,x^2\right)^{3/2}(a+b\,\text{ArcSinh}[c\,x])^n\,dx$$

Optimal (type 4, 616 leaves, 12 steps):

$$-\frac{d\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^{1+n}}{16\,b\,c^3\,(1+n)\sqrt{1+c^2 x^2}}+\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{-\frac{6a}{b}}\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}+$$

$$\frac{2^{-7-2n}\,d\,e^{-\frac{4a}{b}}\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n}\,d\,e^{-\frac{2a}{b}}\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}+$$

$$\frac{2^{-7-n}\,d\,e^{\frac{2a}{b}}\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-2n}\,d\,e^{\frac{4a}{b}}\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{\frac{6a}{b}}\sqrt{d+c^2 d\,x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}$$

Result (type 4, 616 leaves, 13 steps):

$$-\frac{d\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^{1+n}}{16\,b\,c^3\,(1+n)\sqrt{1+c^2 x^2}}+\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{-\frac{6a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}+$$

$$\frac{2^{-7-2n}\,d\,e^{-\frac{4a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n}\,d\,e^{-\frac{2a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}+$$

$$\frac{2^{-7-n}\,d\,e^{\frac{2a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-2n}\,d\,e^{\frac{4a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{\frac{6a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c\,x])^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\sqrt{1+c^2 x^2}}$$

Problem 518: Result optimal but 1 more steps used.

$$\int x\,(d+c^2 d x^2)^{3/2}\,(a+b\,\text{ArcSinh}[c\,x])^n\,dx$$

Optimal (type 4, 542 leaves, 12 steps) :

$$\frac{5^{-1-n}\, d\, e^{-\frac{5a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{3^{-n}\, d\, e^{-\frac{3a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{d\, e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{16\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{d\, e^{a/b} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{16\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{3^{-n}\, d\, e^{\frac{3a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{5^{-1-n}\, d\, e^{\frac{5a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}$$

Result (type 4, 542 leaves, 13 steps):

$$\frac{5^{-1-n}\, d\, e^{-\frac{5a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{3^{-n}\, d\, e^{-\frac{3a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{d\, e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{16\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{d\, e^{a/b} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]}{16\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{3^{-n}\, d\, e^{\frac{3a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{5^{-1-n}\, d\, e^{\frac{5a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{32\, c^2 \sqrt{1+c^2 x^2}}$$

Problem 519: Result optimal but 1 more steps used.

$$\int \left(d+c^2 d x^2\right)^{3/2} (a+b\,\text{ArcSinh}[c x])^n \, dx$$

Optimal (type 4, 420 leaves, 9 steps):

$$\frac{3 d \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^{1+n}}{8 b c (1+n) \sqrt{1+c^2 x^2}}+\frac{2^{-2 (3+n)} d\, e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}+$$
$$\frac{2^{-3-n} d\, e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}-$$
$$\frac{2^{-3-n} d\, e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}-$$
$$\frac{2^{-2 (3+n)} d\, e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}$$

Result (type 4, 420 leaves, 10 steps):

$$\frac{3 d \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^{1+n}}{8 b c (1+n) \sqrt{1+c^2 x^2}}+\frac{2^{-2 (3+n)} d\, e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}+$$
$$\frac{2^{-3-n} d\, e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}-$$
$$\frac{2^{-3-n} d\, e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}-$$
$$\frac{2^{-2 (3+n)} d\, e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c x])^n \left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}$$

Problem 520: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+c^2 d x^2\right)^{3/2} (a+b\,\text{ArcSinh}[c x])^n}{x} \, dx$$

Optimal (type 8, 389 leaves, 15 steps):

$$\frac{3^{-1-n} d^2 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 (a+b \text{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}}+$$

$$\frac{5 d^2 e^{-\frac{a}{b}} \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcSinh}[c x]}{b}\right]}{8 \sqrt{d+c^2 d x^2}}+$$

$$\frac{5 d^2 e^{a/b} \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcSinh}[c x]}{b}\right]}{8 \sqrt{d+c^2 d x^2}}+$$

$$\frac{3^{-1-n} d^2 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 (a+b \text{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}}+d^2 \text{Unintegrable}\left[\frac{(a+b \text{ArcSinh}[c x])^n}{x \sqrt{d+c^2 d x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(d+c^2 d x^2)^{3/2} (a+b \text{ArcSinh}[c x])^n}{x},x\right]$$

Problem 521: Result valid but suboptimal antiderivative.

$$\int \frac{(d+c^2 d x^2)^{3/2} (a+b \text{ArcSinh}[c x])^n}{x^2} \, dx$$

Optimal (type 8, 272 leaves, 9 steps):

$$\frac{3 c d^2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^{1+n}}{2 b (1+n) \sqrt{d+c^2 d x^2}}+\frac{2^{-3-n} c d^2 e^{-\frac{2a}{b}} \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b \text{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}}-$$

$$\frac{2^{-3-n} c d^2 e^{\frac{2a}{b}} \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b \text{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}}+d^2 \text{Unintegrable}\left[\frac{(a+b \text{ArcSinh}[c x])^n}{x^2 \sqrt{d+c^2 d x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(d+c^2 d x^2)^{3/2} (a+b \text{ArcSinh}[c x])^n}{x^2},x\right]$$

Problem 522: Result optimal but 1 more steps used.

$$\int x^2 (d+c^2 d x^2)^{5/2} (a+b \text{ArcSinh}[c x])^n \, dx$$

Optimal (type 4, 816 leaves, 15 steps):

$$-\frac{5\, d^2 \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^{1+n}}{128\, b\, c^3\, (1+n) \sqrt{1+c^2 x^2}}+\frac{2^{-11-3 n} d^2 e^{-\frac{8 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{8 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}+$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{6 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}+$$

$$\frac{2^{-2 (4+n)} d^2 e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}+$$

$$\frac{2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-2 (4+n)} d^2 e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{6 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-11-3 n} d^2 e^{\frac{8 a}{b}} \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{8 (a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}}$$

Result (type 4, 816 leaves, 16 steps) :

$$-\frac{5\,d^2\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^{1+n}}{128\,b\,c^3\,(1+n)\,\sqrt{1+c^2\,x^2}}+\frac{2^{-11-3\,n}\,d^2\,e^{-\frac{8\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{8\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}+$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}+$$

$$\frac{2^{-2\,(4+n)}\,d^2\,e^{-\frac{4\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}-$$

$$\frac{2^{-7-n}\,d^2\,e^{-\frac{2\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}+$$

$$\frac{2^{-7-n}\,d^2\,e^{\frac{2\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}-$$

$$\frac{2^{-2\,(4+n)}\,d^2\,e^{\frac{4\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}-$$

$$\frac{2^{-11-3\,n}\,d^2\,e^{\frac{8\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{8\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]}{c^3\,\sqrt{1+c^2\,x^2}}$$

Problem 523: Result optimal but 1 more steps used.

$$\int x\,\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 745 leaves, 15 steps) :

$$\frac{7^{-1-n} d^2 e^{-\frac{7a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{7 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{5^{-n} d^2 e^{-\frac{5a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{3^{1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{5 d^2 e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{5 d^2 e^{a/b} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{3^{1-n} d^2 e^{\frac{3a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{5^{-n} d^2 e^{\frac{5a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}+$$

$$\frac{7^{-1-n} d^2 e^{\frac{7a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{7 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}$$

Result (type 4, 745 leaves, 16 steps) :

$$\frac{7^{-1-n} d^2 e^{-\frac{7a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{7 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} +$$

$$\frac{5^{-n} d^2 e^{-\frac{5a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{5 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} +$$

$$\frac{3^{1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{3 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} +$$

$$\frac{5 d^2 e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{a+b \text{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} +$$

$$\frac{5 d^2 e^{a/b} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{a+b \text{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} +$$

$$\frac{3^{1-n} d^2 e^{\frac{3a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{3 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} +$$

$$\frac{5^{-n} d^2 e^{\frac{5a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{5 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} +$$

$$\frac{7^{-1-n} d^2 e^{\frac{7a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{7 (a+b \text{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}$$

Problem 524: Result optimal but 1 more steps used.

$$\int (d+c^2 d x^2)^{5/2} (a+b \text{ArcSinh}[c x])^n \, dx$$

Optimal (type 4, 632 leaves, 12 steps):

$$\frac{5 d^2 \sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^{1+n}}{16 b c (1+n)\sqrt{1+c^2 x^2}}+\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{-\frac{6 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}+$$

$$\frac{3\times 2^{-7-2n} d^2 e^{-\frac{4 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}+$$

$$\frac{15\times 2^{-7-n} d^2 e^{-\frac{2 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}-$$

$$\frac{15\times 2^{-7-n} d^2 e^{\frac{2 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}-$$

$$\frac{3\times 2^{-7-2n} d^2 e^{\frac{4 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{\frac{6 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}$$

Result (type 4, 632 leaves, 13 steps):

$$\frac{5 d^2 \sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^{1+n}}{16 b c (1+n)\sqrt{1+c^2 x^2}}+\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{-\frac{6 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}+$$

$$\frac{3\times 2^{-7-2n} d^2 e^{-\frac{4 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}+$$

$$\frac{15\times 2^{-7-n} d^2 e^{-\frac{2 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}-$$

$$\frac{15\times 2^{-7-n} d^2 e^{\frac{2 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}-$$

$$\frac{3\times 2^{-7-2n} d^2 e^{\frac{4 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{\frac{6 a}{b}}\sqrt{d+c^2 d x^2}\,(a+b\,\text{ArcSinh}[c x])^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6(a+b\,\text{ArcSinh}[c x])}{b}\right]}{c\sqrt{1+c^2 x^2}}$$

Problem 525: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d + c^2 d x^2\right)^{5/2} \left(a + b \operatorname{ArcSinh}[c x]\right)^n}{x} \, dx$$

Optimal (type 8, 755 leaves, 27 steps):

$$\frac{5^{-1-n} d^3 e^{-\frac{5a}{b}} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} -$$
$$\frac{5 \times 3^{-1-n} d^3 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} +$$
$$\frac{3^{-n} d^3 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}} +$$
$$\frac{11 d^3 e^{-\frac{a}{b}} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 \sqrt{d+c^2 d x^2}} +$$
$$\frac{11 d^3 e^{a/b} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 \sqrt{d+c^2 d x^2}} -$$
$$\frac{5 \times 3^{-1-n} d^3 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} +$$
$$\frac{3^{-n} d^3 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}} +$$
$$\frac{5^{-1-n} d^3 e^{\frac{5a}{b}} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} + d^3 \operatorname{Unintegrable}\left[\frac{\left(a+b \operatorname{ArcSinh}[c x]\right)^n}{x \sqrt{d+c^2 d x^2}}, x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\left(d+c^2 d x^2\right)^{5/2} \left(a+b \operatorname{ArcSinh}[c x]\right)^n}{x}, x\right]$$

Problem 526: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+c^2 d x^2\right)^{5/2} (a+b \operatorname{ArcSinh}[c x])^n}{x^2} \, dx$$

Optimal (type 8, 454 leaves, 18 steps):

$$\frac{15 c d^3 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{8 b (1+n) \sqrt{d+c^2 d x^2}}+\frac{2^{-2 (3+n)} c d^3 e^{-\frac{4 a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{4 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}}+$$
$$\frac{2^{-2-n} c d^3 e^{-\frac{2 a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{2 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}}-$$
$$\frac{2^{-2-n} c d^3 e^{\frac{2 a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{2 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}}-$$
$$\frac{2^{-2 (3+n)} c d^3 e^{\frac{4 a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{4 (a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}}+d^3 \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcSinh}[c x])^n}{x^2 \sqrt{d+c^2 d x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\left(d+c^2 d x^2\right)^{5/2} (a+b \operatorname{ArcSinh}[c x])^n}{x^2},x\right]$$

Test results for the 371 problems in "7.1.5 Inverse hyperbolic sine functions.m"

Problem 50: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{\sqrt{d+c^2 d x^2}} \, dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 b c \sqrt{d+c^2 d x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 170: Result valid but suboptimal antiderivative.

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{ArcSinh}[c+d\,x])^3}\,dx$$

Optimal (type 4, 246 leaves, 18 steps):

$$-\frac{e^2\,(c+d\,x)^2\,\sqrt{1+(c+d\,x)^2}}{2\,b\,d\,(a+b\,\text{ArcSinh}[c+d\,x])^2}-\frac{e^2\,(c+d\,x)}{b^2\,d\,(a+b\,\text{ArcSinh}[c+d\,x])}-\frac{3\,e^2\,(c+d\,x)^3}{2\,b^2\,d\,(a+b\,\text{ArcSinh}[c+d\,x])}-\frac{e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcSinh}[c+d\,x]}{b}\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcSinh}[c+d\,x])}{b}\right]}{8\,b^3\,d}+\frac{e^2\,\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcSinh}[c+d\,x]}{b}\right]}{8\,b^3\,d}-\frac{9\,e^2\,\text{Sinh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcSinh}[c+d\,x])}{b}\right]}{8\,b^3\,d}$$

Result (type 4, 305 leaves, 18 steps):

$$-\frac{e^2\,(c+d\,x)^2\,\sqrt{1+(c+d\,x)^2}}{2\,b\,d\,(a+b\,\text{ArcSinh}[c+d\,x])^2}-\frac{e^2\,(c+d\,x)}{b^2\,d\,(a+b\,\text{ArcSinh}[c+d\,x])}-\frac{3\,e^2\,(c+d\,x)^3}{2\,b^2\,d\,(a+b\,\text{ArcSinh}[c+d\,x])}-\frac{9\,e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c+d\,x]\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcSinh}[c+d\,x]\right]}{8\,b^3\,d}+\frac{e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcSinh}[c+d\,x]}{b}\right]}{b^3\,d}+\frac{9\,e^2\,\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c+d\,x]\right]}{8\,b^3\,d}-\frac{9\,e^2\,\text{Sinh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcSinh}[c+d\,x]\right]}{8\,b^3\,d}-\frac{e^2\,\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcSinh}[c+d\,x]}{b}\right]}{b^3\,d}$$

Problem 369: Unable to integrate problem.

$$\int \frac{x}{\text{ArcSinh}[\text{Sinh}[x]]}\,dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\text{ArcSinh}[\text{Sinh}[x]]+\text{Log}[\text{ArcSinh}[\text{Sinh}[x]]]\,\left(-\text{ArcSinh}[\text{Sinh}[x]]+x\,\sqrt{\text{Cosh}[x]^2}\,\text{Sech}[x]\right)$$

Result (type 8, 9 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{x}{\text{ArcSinh}[\text{Sinh}[x]]},x\right]$$

Test results for the 166 problems in "7.2.2 (d x)^m (a+b arccosh(c x))^n.m"

Problem 163: Result valid but suboptimal antiderivative.

$$\int \sqrt{f x}\,(a+b\,\text{ArcCosh}[c x])^2\,dx$$

Optimal (type 5, 128 leaves, 2 steps):

$$\frac{2 (f x)^{3/2} (a+b\,\text{ArcCosh}[c x])^2}{3 f}-\frac{8 b c (f x)^{5/2} \sqrt{1-c x}\,(a+b\,\text{ArcCosh}[c x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2 x^2\right]}{15 f^2 \sqrt{-1+c x}}-\frac{16 b^2 c^2 (f x)^{7/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2 x^2\right]}{105 f^3}$$

Result (type 5, 141 leaves, 2 steps):

$$\frac{2 (f x)^{3/2} (a+b\,\text{ArcCosh}[c x])^2}{3 f}-\frac{8 b c (f x)^{5/2} \sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2 x^2\right]}{15 f^2 \sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{16 b^2 c^2 (f x)^{7/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2 x^2\right]}{105 f^3}$$

Test results for the 569 problems in "7.2.4 (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n.m"

Problem 58: Result optimal but 1 more steps used.

$$\int x^4 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,dx$$

Optimal (type 3, 278 leaves, 7 steps):

$$\frac{b x^2 \sqrt{d-c^2 d x^2}}{32 c^3 \sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{b x^4 \sqrt{d-c^2 d x^2}}{96 c \sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{b c x^6 \sqrt{d-c^2 d x^2}}{36 \sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{x \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{16 c^4}-\frac{x^3 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{24 c^2}+\frac{1}{6} x^5 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])-\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2}{32 b c^5 \sqrt{-1+c x}\,\sqrt{1+c x}}$$

Result (type 3, 278 leaves, 8 steps):

$$\frac{b x^2 \sqrt{d - c^2 d x^2}}{32 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b x^4 \sqrt{d - c^2 d x^2}}{96 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^6 \sqrt{d - c^2 d x^2}}{36 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{x \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{16 c^4} - \frac{x^3 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{24 c^2} + \frac{1}{6} x^5 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) - \frac{\sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{32 b c^5 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 59: Result optimal but 1 more steps used.

$$\int x^2 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) dx$$

Optimal (type 3, 201 leaves, 5 steps):

$$\frac{b x^2 \sqrt{d - c^2 d x^2}}{16 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^4 \sqrt{d - c^2 d x^2}}{16 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{x \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{8 c^2} + \frac{1}{4} x^3 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) - \frac{\sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{16 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 201 leaves, 6 steps):

$$\frac{b x^2 \sqrt{d - c^2 d x^2}}{16 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^4 \sqrt{d - c^2 d x^2}}{16 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{x \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{8 c^2} + \frac{1}{4} x^3 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) - \frac{\sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{16 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 60: Result optimal but 1 more steps used.

$$\int \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) dx$$

Optimal (type 3, 124 leaves, 3 steps):

$$-\frac{b c x^2 \sqrt{d - c^2 d x^2}}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{1}{2} x \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) - \frac{\sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{4 b c \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 124 leaves, 4 steps):

$$-\frac{b c x^2 \sqrt{d - c^2 d x^2}}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{1}{2} x \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) - \frac{\sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{4 b c \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 61: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^2}\,dx$$

Optimal (type 3, 118 leaves, 3 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x}+\frac{c\,\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c\,\sqrt{d-c^2 d x^2}\,\text{Log}[x]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 118 leaves, 4 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x}+\frac{c\,\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c\,\sqrt{d-c^2 d x^2}\,\text{Log}[x]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 63: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^6}\,dx$$

Optimal (type 3, 199 leaves, 4 steps):

$$-\frac{b\,c\,\sqrt{d-c^2 d x^2}}{20\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,\sqrt{d-c^2 d x^2}}{30\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2 d x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,d\,x^5}-\frac{2\,c^2\,\left(d-c^2 d x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{15\,d\,x^3}-\frac{2\,b\,c^5\,\sqrt{d-c^2 d x^2}\,\text{Log}[x]}{15\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 226 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d-c^2 d x^2}}{20\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,\sqrt{d-c^2 d x^2}}{30\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,x^5}+\frac{c^2\,\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{15\,x^3}+\frac{2\,c^4\,\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{15\,x}-\frac{2\,b\,c^5\,\sqrt{d-c^2 d x^2}\,\text{Log}[x]}{15\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 64: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^8}\,dx$$

Optimal (type 3, 279 leaves, 4 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{42 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2}}{140 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^5 \sqrt{d-c^2 d x^2}}{105 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])}{7 d x^7}-\frac{4 c^2 \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])}{35 d x^5}-\frac{8 c^4 \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])}{105 d x^3}-\frac{8 b c^7 \sqrt{d-c^2 d x^2} \text{Log}[x]}{105 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 303 leaves, 5 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{42 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2}}{140 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^5 \sqrt{d-c^2 d x^2}}{105 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{7 x^7}+\frac{c^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{35 x^5}+\frac{4 c^4 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{105 x^3}+\frac{8 c^6 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{105 x}-\frac{8 b c^7 \sqrt{d-c^2 d x^2} \text{Log}[x]}{105 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x^5 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 272 leaves, 3 steps):

$$\frac{8 b x \sqrt{d-c^2 d x^2}}{105 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{4 b x^3 \sqrt{d-c^2 d x^2}}{315 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b x^5 \sqrt{d-c^2 d x^2}}{175 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c x^7 \sqrt{d-c^2 d x^2}}{49 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])}{3 c^6 d}+\frac{2 \left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{5 c^6 d^2}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{7 c^6 d^3}$$

Result (type 3, 302 leaves, 4 steps):

$$\frac{8 b x \sqrt{d-c^2 d x^2}}{105 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{4 b x^3 \sqrt{d-c^2 d x^2}}{315 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b x^5 \sqrt{d-c^2 d x^2}}{175 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c x^7 \sqrt{d-c^2 d x^2}}{49 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{8 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{105 c^6}-\frac{4 x^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{35 c^4}-\frac{x^4 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{7 c^2}$$

Problem 66: Result valid but suboptimal antiderivative.

$$\int x^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$\frac{2\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^3\,\sqrt{d-c^2\,d\,x^2}}{45\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^5\,\sqrt{d-c^2\,d\,x^2}}{25\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^4\,d}+\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^4\,d^2}$$

Result (type 3, 214 leaves, 4 steps):

$$\frac{2\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^3\,\sqrt{d-c^2\,d\,x^2}}{45\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^5\,\sqrt{d-c^2\,d\,x^2}}{25\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{15\,c^4}-\frac{x^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^2}$$

Problem 68: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x}\,dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$-\frac{b\,c\,x\,\sqrt{d-c^2\,d\,x^2}}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)-\frac{2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 213 leaves, 9 steps):

$$-\frac{b\,c\,x\,\sqrt{d-c^2\,d\,x^2}}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)-\frac{2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 69: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^3}\,dx$$

Optimal (type 4, 235 leaves, 8 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{2 x \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{2 x^2}+\frac{c^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{i b c^2 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{i b c^2 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{2 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 235 leaves, 9 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{2 x \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{2 x^2}+\frac{c^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{i b c^2 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{i b c^2 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{2 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 70: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{x^5} \, dx$$

Optimal (type 4, 315 leaves, 10 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{12 x^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2}}{8 x \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{4 x^4}+\frac{c^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{8 x^2}+\frac{c^4 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{i b c^4 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{i b c^4 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 315 leaves, 11 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{12 x^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2}}{8 x \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{4 x^4}+\frac{c^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{8 x^2}+\frac{c^4 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{i b c^4 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{i b c^4 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 77: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{x^8} \, dx$$

Optimal (type 3, 247 leaves, 5 steps):

$$-\frac{b c d \sqrt{d-c^2 d x^2}}{42 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^3 d \sqrt{d-c^2 d x^2}}{35 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d \sqrt{d-c^2 d x^2}}{70 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{7 d x^7}-\frac{2 c^2 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{35 d x^5}+\frac{2 b c^7 d \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{35 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 322 leaves, 6 steps):

$$-\frac{b c d \sqrt{d-c^2 d x^2}}{42 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^3 d \sqrt{d-c^2 d x^2}}{35 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d \sqrt{d-c^2 d x^2}}{70 x^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{3 c^2 d \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{35 x^5}-\frac{c^4 d \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{35 x^3}-\frac{2 c^6 d \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{35 x}-\frac{d (1-c x) (1+c x) \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{7 x^7}+\frac{2 b c^7 d \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{35 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{x^{10}} \, dx$$

Optimal (type 3, 328 leaves, 5 steps):

$$-\frac{b c d \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b c^3 d \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d \sqrt{d-c^2 d x^2}}{420 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b c^7 d \sqrt{d-c^2 d x^2}}{315 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{9 d x^9}-\frac{4 c^2 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{63 d x^7}-\frac{8 c^4 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \operatorname{ArcCosh}[c x]\right)}{315 d x^5}+\frac{8 b c^9 d \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{315 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 401 leaves, 6 steps):

$$-\frac{b\,c\,d\,\sqrt{d-c^2\,d\,x^2}}{72\,x^8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5\,b\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}}{189\,x^6\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}{420\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2\,b\,c^7\,d\,\sqrt{d-c^2\,d\,x^2}}{315\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c^2\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{21\,x^7}-\frac{c^4\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{105\,x^5}-\frac{4\,c^6\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{315\,x^3}-\frac{8\,c^8\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{315\,x}-\frac{d\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{9\,x^9}+\frac{8\,b\,c^9\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{315\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 79: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^{12}}\,dx$$

Optimal (type 3, 409 leaves, 5 steps):

$$-\frac{b\,c\,d\,\sqrt{d-c^2\,d\,x^2}}{110\,x^{10}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}}{66\,x^8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}{1386\,x^6\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^7\,d\,\sqrt{d-c^2\,d\,x^2}}{770\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{4\,b\,c^9\,d\,\sqrt{d-c^2\,d\,x^2}}{1155\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{11\,d\,x^{11}}-\frac{2\,c^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{33\,d\,x^9}-\frac{8\,c^4\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{231\,d\,x^7}-\frac{16\,c^6\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{1155\,d\,x^5}+\frac{16\,b\,c^{11}\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{1155\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 480 leaves, 6 steps):

$$-\frac{b\,c\,d\,\sqrt{d-c^2\,d\,x^2}}{110\,x^{10}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}}{66\,x^8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}{1386\,x^6\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^7\,d\,\sqrt{d-c^2\,d\,x^2}}{770\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{4\,b\,c^9\,d\,\sqrt{d-c^2\,d\,x^2}}{1155\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c^2\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{33\,x^9}-\frac{c^4\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{231\,x^7}-\frac{2\,c^6\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{385\,x^5}-\frac{8\,c^8\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{1155\,x^3}-\frac{16\,c^{10}\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{1155\,x}-\frac{d\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{11\,x^{11}}+\frac{16\,b\,c^{11}\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{1155\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 80: Result valid but suboptimal antiderivative.

$$\int x^7\,\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,dx$$

Optimal (type 3, 399 leaves, 4 steps):

$$\frac{16\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{1155\,c^7\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{8\,b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{3465\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2\,b\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{1925\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{1617\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{4\,b\,c\,d\,x^9\,\sqrt{d-c^2\,d\,x^2}}{297\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^{11}\,\sqrt{d-c^2\,d\,x^2}}{121\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^8\,d}+\frac{3\,\left(d-c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,c^8\,d^2}-\frac{\left(d-c^2\,d\,x^2\right)^{9/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^8\,d^3}+\frac{\left(d-c^2\,d\,x^2\right)^{11/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{11\,c^8\,d^4}$$

Result (type 3, 460 leaves, 5 steps):

$$\frac{16\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{1155\,c^7\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{8\,b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{3465\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2\,b\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{1925\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{1617\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{4\,b\,c\,d\,x^9\,\sqrt{d-c^2\,d\,x^2}}{297\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^{11}\,\sqrt{d-c^2\,d\,x^2}}{121\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{16\,d\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{1155\,c^8}-\frac{8\,d\,x^2\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{231\,c^6}-\frac{2\,d\,x^4\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{33\,c^4}-\frac{d\,x^6\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{11\,c^2}$$

Problem 81: Result valid but suboptimal antiderivative.

$$\int x^5\,\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,dx$$

Optimal (type 3, 321 leaves, 4 steps):

$$\frac{8\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{315\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{945\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{525\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{10\,b\,c\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{441\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^9\,\sqrt{d-c^2\,d\,x^2}}{81\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^6\,d}+\frac{2\,\left(d-c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,c^6\,d^2}-\frac{\left(d-c^2\,d\,x^2\right)^{9/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{9\,c^6\,d^3}$$

Result (type 3, 366 leaves, 5 steps):

$$\frac{8\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{315\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{945\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{525\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{10\,b\,c\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{441\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^9\,\sqrt{d-c^2\,d\,x^2}}{81\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{8\,d\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{315\,c^6}-\frac{4\,d\,x^2\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{63\,c^4}-\frac{d\,x^4\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{9\,c^2}$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 243 leaves, 4 steps):

$$\frac{2 b d x \sqrt{d-c^2 d x^2}}{35 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d x^3 \sqrt{d-c^2 d x^2}}{105 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{8 b c d x^5 \sqrt{d-c^2 d x^2}}{175 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 d x^7 \sqrt{d-c^2 d x^2}}{49 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{5 c^4 d}+\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{7 c^4 d^2}$$

Result (type 3, 272 leaves, 5 steps):

$$\frac{2 b d x \sqrt{d-c^2 d x^2}}{35 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d x^3 \sqrt{d-c^2 d x^2}}{105 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{8 b c d x^5 \sqrt{d-c^2 d x^2}}{175 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 d x^7 \sqrt{d-c^2 d x^2}}{49 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 d (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{35 c^4}-\frac{d x^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{7 c^2}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int \left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 293 leaves, 10 steps):

$$-\frac{25 b c d^2 x^2 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b c^3 d^2 x^4 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d^2 \left(1-c^2 x^2\right)^3 \sqrt{d-c^2 d x^2}}{36 c \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])+\frac{5}{24} d x \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])+\frac{1}{6} x \left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])-\frac{5 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{32 b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 324 leaves, 9 steps):

$$-\frac{25 b c d^2 x^2 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b c^3 d^2 x^4 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d^2 \left(1-c^2 x^2\right)^3 \sqrt{d-c^2 d x^2}}{36 c \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])+\frac{5}{24} d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])+\frac{1}{6} d^2 x (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])-\frac{5 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{32 b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 90: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{x^2} \, dx$$

Optimal (type 3, 284 leaves, 12 steps):

$$\frac{9 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^4 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])-\frac{5}{4} c^2 d x \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])-\frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{x}+\frac{15 c d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{16 b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 315 leaves, 11 steps):

$$\frac{9 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^4 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])-\frac{5}{4} c^2 d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])-\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{x}+\frac{15 c d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{16 b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 91: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{x^4} \, dx$$

Optimal (type 3, 293 leaves, 12 steps):

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2}}{6 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^2 \sqrt{d-c^2 d x^2}}{4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5}{2} c^4 d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])+\frac{5 c^2 d \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])}{3 x}-\frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{3 x^3}-\frac{5 c^3 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{4 b \sqrt{-1+c x} \sqrt{1+c x}}-\frac{7 b c^3 d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 324 leaves, 11 steps):

$$-\frac{b\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}}{6\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5}{2}\,c^4\,d^2\,x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)+$$
$$\frac{5\,c^2\,d^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x}-\frac{d^2\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^3}-$$
$$\frac{5\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{4\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{7\,b\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 92: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^6}\,dx$$

Optimal (type 3, 293 leaves, 12 steps):

$$-\frac{b\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}}{20\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{11\,b\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}}{30\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{c^4\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x}+\frac{c^2\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^3}-$$
$$\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,x^5}+\frac{c^5\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{23\,b\,c^5\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{15\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 324 leaves, 11 steps):

$$-\frac{b\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}}{20\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{11\,b\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}}{30\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{c^4\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x}+$$
$$\frac{c^2\,d^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^3}-\frac{d^2\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,x^5}+$$
$$\frac{c^5\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{23\,b\,c^5\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{15\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 94: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^{10}}\,dx$$

Optimal (type 3, 314 leaves, 6 steps):

$$-\frac{b c^3 d^2 \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^5 d^2 \sqrt{d-c^2 d x^2}}{42 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{21 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d^2 \left(1-c^2 x^2\right)^4 \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{9 d x^9}-\frac{2 c^2 \left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{63 d x^7}-\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2} \text{ Log}[x]}{63 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 448 leaves, 7 steps):

$$-\frac{b c^3 d^2 \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^5 d^2 \sqrt{d-c^2 d x^2}}{42 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{21 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d^2 \left(1-c^2 x^2\right)^4 \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{c^4 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{21 x^5}+\frac{c^6 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{63 x^3}+\frac{2 c^8 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{63 x}+\frac{5 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{63 x^7}-\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{9 x^9}-\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2} \text{ Log}[x]}{63 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 95: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{x^{12}} \, dx$$

Optimal (type 3, 385 leaves, 5 steps):

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2}}{110 x^{10} \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^3 d^2 \sqrt{d-c^2 d x^2}}{792 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{113 b c^5 d^2 \sqrt{d-c^2 d x^2}}{4158 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{924 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2}}{693 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{11 d x^{11}}-\frac{4 c^2 \left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{99 d x^9}-\frac{8 c^4 \left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{693 d x^7}-\frac{8 b c^{11} d^2 \sqrt{d-c^2 d x^2} \text{ Log}[x]}{693 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 519 leaves, 6 steps):

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2}}{110 x^{10} \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^3 d^2 \sqrt{d-c^2 d x^2}}{792 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{113 b c^5 d^2 \sqrt{d-c^2 d x^2}}{4158 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{924 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2}}{693 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{5 c^4 d^2 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{231 x^7}+\frac{c^6 d^2 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{231 x^5}+\frac{4 c^8 d^2 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{693 x^3}+\frac{8 c^{10} d^2 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{693 x}+\frac{5 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{99 x^9}-\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{11 x^{11}}-\frac{8 b c^{11} d^2 \sqrt{d-c^2 d x^2}\, \text{Log}[x]}{693 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 96: Result valid but suboptimal antiderivative.

$$\int x^7 \left(d-c^2 d x^2\right)^{5/2} (a+b \,\text{ArcCosh}[c x])\, dx$$

Optimal (type 3, 458 leaves, 4 steps):

$$\frac{16 b d^2 x \sqrt{d-c^2 d x^2}}{3003 c^7 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{8 b d^2 x^3 \sqrt{d-c^2 d x^2}}{9009 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b d^2 x^5 \sqrt{d-c^2 d x^2}}{5005 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b d^2 x^7 \sqrt{d-c^2 d x^2}}{21021 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{53 b c d^2 x^9 \sqrt{d-c^2 d x^2}}{3861 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{27 b c^3 d^2 x^{11} \sqrt{d-c^2 d x^2}}{1573 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^{13} \sqrt{d-c^2 d x^2}}{169 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \,\text{ArcCosh}[c x])}{7 c^8 d}+\frac{\left(d-c^2 d x^2\right)^{9/2} (a+b \,\text{ArcCosh}[c x])}{3 c^8 d^2}-\frac{3 \left(d-c^2 d x^2\right)^{11/2} (a+b \,\text{ArcCosh}[c x])}{11 c^8 d^3}+\frac{\left(d-c^2 d x^2\right)^{13/2} (a+b \,\text{ArcCosh}[c x])}{13 c^8 d^4}$$

Result (type 3, 527 leaves, 5 steps):

$$\frac{16 b d^2 x \sqrt{d-c^2 d x^2}}{3003 c^7 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{8 b d^2 x^3 \sqrt{d-c^2 d x^2}}{9009 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b d^2 x^5 \sqrt{d-c^2 d x^2}}{5005 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b d^2 x^7 \sqrt{d-c^2 d x^2}}{21021 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{53 b c d^2 x^9 \sqrt{d-c^2 d x^2}}{3861 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{27 b c^3 d^2 x^{11} \sqrt{d-c^2 d x^2}}{1573 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^{13} \sqrt{d-c^2 d x^2}}{169 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{16 d^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{3003 c^8}-\frac{8 d^2 x^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{429 c^6}-\frac{6 d^2 x^4 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{143 c^4}-\frac{d^2 x^6 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2}\, (a+b \,\text{ArcCosh}[c x])}{13 c^2}$$

Problem 97: Result valid but suboptimal antiderivative.

$$\int x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 378 leaves, 4 steps):

$$\frac{8 b d^2 x \sqrt{d-c^2 d x^2}}{693 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{4 b d^2 x^3 \sqrt{d-c^2 d x^2}}{2079 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d^2 x^5 \sqrt{d-c^2 d x^2}}{1155 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{113 b c d^2 x^7 \sqrt{d-c^2 d x^2}}{4851 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^3 d^2 x^9 \sqrt{d-c^2 d x^2}}{891 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^{11} \sqrt{d-c^2 d x^2}}{121 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{7 c^6 d}+\frac{2 \left(d-c^2 d x^2\right)^{9/2} (a+b \text{ArcCosh}[c x])}{9 c^6 d^2}-\frac{\left(d-c^2 d x^2\right)^{11/2} (a+b \text{ArcCosh}[c x])}{11 c^6 d^3}$$

Result (type 3, 429 leaves, 5 steps):

$$\frac{8 b d^2 x \sqrt{d-c^2 d x^2}}{693 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{4 b d^2 x^3 \sqrt{d-c^2 d x^2}}{2079 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d^2 x^5 \sqrt{d-c^2 d x^2}}{1155 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{113 b c d^2 x^7 \sqrt{d-c^2 d x^2}}{4851 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^3 d^2 x^9 \sqrt{d-c^2 d x^2}}{891 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^{11} \sqrt{d-c^2 d x^2}}{121 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{8 d^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{693 c^6}-\frac{4 d^2 x^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{99 c^4}-\frac{d^2 x^4 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{11 c^2}$$

Problem 98: Result valid but suboptimal antiderivative.

$$\int x^3 \left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 298 leaves, 4 steps):

$$\frac{2 b d^2 x \sqrt{d-c^2 d x^2}}{63 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d^2 x^3 \sqrt{d-c^2 d x^2}}{189 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d^2 x^5 \sqrt{d-c^2 d x^2}}{21 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{19 b c^3 d^2 x^7 \sqrt{d-c^2 d x^2}}{441 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^9 \sqrt{d-c^2 d x^2}}{81 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{7 c^4 d}+\frac{\left(d-c^2 d x^2\right)^{9/2} (a+b \text{ArcCosh}[c x])}{9 c^4 d^2}$$

Result (type 3, 331 leaves, 5 steps):

$$\frac{2 b d^2 x \sqrt{d-c^2 d x^2}}{63 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d^2 x^3 \sqrt{d-c^2 d x^2}}{189 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d^2 x^5 \sqrt{d-c^2 d x^2}}{21 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{19 b c^3 d^2 x^7 \sqrt{d-c^2 d x^2}}{441 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^9 \sqrt{d-c^2 d x^2}}{81 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 d^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{63 c^4}-\frac{d^2 x^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{9 c^2}$$

Problem 103: Result valid but suboptimal antiderivative.

$$\int \sqrt{1-x^2}\ \mathrm{ArcCosh}[x]\, dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$-\frac{\sqrt{1-x}\ x^2}{4\sqrt{-1+x}}+\frac{1}{2}x\sqrt{1-x^2}\ \mathrm{ArcCosh}[x]-\frac{\sqrt{1-x}\ \mathrm{ArcCosh}[x]^2}{4\sqrt{-1+x}}$$

Result (type 3, 84 leaves, 4 steps):

$$-\frac{x^2\sqrt{1-x^2}}{4\sqrt{-1+x}\ \sqrt{1+x}}+\frac{1}{2}x\sqrt{1-x^2}\ \mathrm{ArcCosh}[x]-\frac{\sqrt{1-x^2}\ \mathrm{ArcCosh}[x]^2}{4\sqrt{-1+x}\ \sqrt{1+x}}$$

Problem 104: Result valid but suboptimal antiderivative.

$$\int \frac{x^5\ (a+b\,\mathrm{ArcCosh}[c\,x])}{\sqrt{d-c^2\,d\,x^2}}\, dx$$

Optimal (type 3, 236 leaves, 6 steps):

$$-\frac{8\,b\,x\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{15\,c^5\sqrt{d-c^2\,d\,x^2}}-\frac{4\,b\,x^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{45\,c^3\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^5\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{25\,c\sqrt{d-c^2\,d\,x^2}}-\frac{8\sqrt{d-c^2\,d\,x^2}\ (a+b\,\mathrm{ArcCosh}[c\,x])}{15\,c^6\,d}-\frac{4\,x^2\sqrt{d-c^2\,d\,x^2}\ (a+b\,\mathrm{ArcCosh}[c\,x])}{15\,c^4\,d}-\frac{x^4\sqrt{d-c^2\,d\,x^2}\ (a+b\,\mathrm{ArcCosh}[c\,x])}{5\,c^2\,d}$$

Result (type 3, 260 leaves, 7 steps):

$$-\frac{8\,b\,x\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{15\,c^5\sqrt{d-c^2\,d\,x^2}}-\frac{4\,b\,x^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{45\,c^3\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^5\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{25\,c\sqrt{d-c^2\,d\,x^2}}-\frac{8\,(1-c\,x)\,(1+c\,x)\,(a+b\,\mathrm{ArcCosh}[c\,x])}{15\,c^6\sqrt{d-c^2\,d\,x^2}}-\frac{4\,x^2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\mathrm{ArcCosh}[c\,x])}{15\,c^4\sqrt{d-c^2\,d\,x^2}}-\frac{x^4\,(1-c\,x)\,(1+c\,x)\,(a+b\,\mathrm{ArcCosh}[c\,x])}{5\,c^2\sqrt{d-c^2\,d\,x^2}}$$

Problem 105: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\ (a+b\,\mathrm{ArcCosh}[c\,x])}{\sqrt{d-c^2\,d\,x^2}}\, dx$$

Optimal (type 3, 212 leaves, 5 steps):

$$-\frac{3\,b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{3\,x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{8\,c^4\,d}-\frac{x^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,c^2\,d}+\frac{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{16\,b\,c^5\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 228 leaves, 6 steps):

$$-\frac{3\,b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{3\,x\,(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{x^3\,(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,c^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{16\,b\,c^5\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 106: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 156 leaves, 4 steps):

$$-\frac{2\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{3\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{9\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^4\,d}-\frac{x^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^2\,d}$$

Result (type 3, 172 leaves, 5 steps):

$$-\frac{2\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{3\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{9\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{x^2\,(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 107: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 132 leaves, 3 steps):

$$-\frac{b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{4\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,c^2\,d}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 140 leaves, 4 steps):

$$-\frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x}}{4 c \sqrt{d-c^2 d x^2}}-\frac{x (1-c x) (1+c x) (a+b \text{ArcCosh}[c x])}{2 c^2 \sqrt{d-c^2 d x^2}}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{4 b c^3 \sqrt{d-c^2 d x^2}}$$

Problem 108: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \text{ArcCosh}[c x])}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$-\frac{b x \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{d-c^2 d x^2}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{c^2 d}$$

Result (type 3, 80 leaves, 3 steps):

$$-\frac{b x \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{d-c^2 d x^2}}-\frac{(1-c x) (1+c x) (a+b \text{ArcCosh}[c x])}{c^2 \sqrt{d-c^2 d x^2}}$$

Problem 109: Result optimal but 1 more steps used.

$$\int \frac{a+b \text{ArcCosh}[c x]}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{2 b c \sqrt{d-c^2 d x^2}}$$

Result (type 3, 53 leaves, 2 steps):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{2 b c \sqrt{d-c^2 d x^2}}$$

Problem 110: Result optimal but 1 more steps used.

$$\int \frac{a+b \text{ArcCosh}[c x]}{x \sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 4, 151 leaves, 6 steps):

$$\frac{2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 151 leaves, 7 steps):

$$\frac{2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 111: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^2\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$-\frac{\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{d\,x}-\frac{b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 79 leaves, 3 steps):

$$-\frac{(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{x\sqrt{d-c^2\,d\,x^2}}-\frac{b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 112: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^3\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 238 leaves, 8 steps):

$$\frac{b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{2\,x\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2\,d\,x^2}+\frac{c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 246 leaves, 9 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{2\,x\,\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{2\,x^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 113: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^4\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,x^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{3\,d\,x^3}-\frac{2\,c^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{3\,d\,x}-\frac{2\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{3\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 171 leaves, 5 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,x^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{3\,x^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,c^2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{3\,x\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{3\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 114: Result valid but suboptimal antiderivative.

$$\int\frac{x^5\,(a+b\,\text{ArcCosh}[c\,x])}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 233 leaves, 5 steps):

$$-\frac{5\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{3\,c^5\,d^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,x^3\,\sqrt{d-c^2\,d\,x^2}}{9\,c^3\,d^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{a+b\,\text{ArcCosh}[c\,x]}{c^6\,d\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{c^6\,d^2}-\frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^6\,d^3}-\frac{b\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}[c\,x]}{c^6\,d^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 262 leaves, 5 steps):

$$\frac{5\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{3\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{9\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}}+\frac{x^4\,(a+b\,\text{ArcCosh}[c\,x])}{c^2\,d\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{8\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^6\,d\,\sqrt{d-c^2\,d\,x^2}}+\frac{4\,x^2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^4\,d\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTanh}[c\,x]}{c^6\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \left(a + b\,\text{ArcCosh}[c\,x]\right)}{\left(d - c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 226 leaves, 7 steps):

$$\frac{b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{4\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}} + \frac{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{c^2\,d\,\sqrt{d-c^2\,d\,x^2}} + \frac{3\,x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,c^4\,d^2} - \frac{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{4\,b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}} - \frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 237 leaves, 8 steps):

$$\frac{b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{4\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}} + \frac{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{c^2\,d\,\sqrt{d-c^2\,d\,x^2}} + \frac{3\,x\,(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,c^4\,d\,\sqrt{d-c^2\,d\,x^2}} - \frac{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{4\,b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}} - \frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 119: Result optimal but 1 more steps used.

$$\int \frac{a + b\,\text{ArcCosh}[c\,x]}{\left(d - c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 84 leaves, 2 steps):

$$\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d\,\sqrt{d-c^2\,d\,x^2}} - \frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 84 leaves, 3 steps):

$$\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d\,\sqrt{d-c^2\,d\,x^2}} - \frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{a + b\,\text{ArcCosh}[c\,x]}{x^4\,\left(d - c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{6 d^2 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{a+b \operatorname{ArcCosh}[c x]}{3 d x^3 \sqrt{d-c^2 d x^2}}-\frac{4 c^2 (a+b \operatorname{ArcCosh}[c x])}{3 d x \sqrt{d-c^2 d x^2}}+\frac{8 c^4 x (a+b \operatorname{ArcCosh}[c x])}{3 d \sqrt{d-c^2 d x^2}}+\frac{5 b c^3 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{3 d^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2} \operatorname{Log}[1-c^2 x^2]}{2 d^2 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 250 leaves, 6 steps):

$$\frac{b c \sqrt{-1+c x} \sqrt{1+c x}}{6 d x^2 \sqrt{d-c^2 d x^2}}-\frac{a+b \operatorname{ArcCosh}[c x]}{3 d x^3 \sqrt{d-c^2 d x^2}}-\frac{4 c^2 (a+b \operatorname{ArcCosh}[c x])}{3 d x \sqrt{d-c^2 d x^2}}+\frac{8 c^4 x (a+b \operatorname{ArcCosh}[c x])}{3 d \sqrt{d-c^2 d x^2}}-\frac{5 b c^3 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{Log}[x]}{3 d \sqrt{d-c^2 d x^2}}-\frac{b c^3 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{Log}[1-c^2 x^2]}{2 d \sqrt{d-c^2 d x^2}}$$

Problem 124: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a+b \operatorname{ArcCosh}[c x])}{(d-c^2 d x^2)^{5/2}} \, dx$$

Optimal (type 3, 243 leaves, 5 steps):

$$\frac{b x \sqrt{d-c^2 d x^2}}{c^5 d^3 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b x \sqrt{d-c^2 d x^2}}{6 c^5 d^3 \sqrt{-1+c x} \sqrt{1+c x} (1-c^2 x^2)}+\frac{a+b \operatorname{ArcCosh}[c x]}{3 c^6 d (d-c^2 d x^2)^{3/2}}-\frac{2 (a+b \operatorname{ArcCosh}[c x])}{c^6 d^2 \sqrt{d-c^2 d x^2}}-\frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{c^6 d^3}+\frac{11 b \sqrt{d-c^2 d x^2} \operatorname{ArcTanh}[c x]}{6 c^6 d^3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 280 leaves, 6 steps):

$$-\frac{b x \sqrt{-1+c x} \sqrt{1+c x}}{c^5 d^2 \sqrt{d-c^2 d x^2}}+\frac{b x \sqrt{-1+c x} \sqrt{1+c x}}{6 c^5 d^2 (1-c^2 x^2) \sqrt{d-c^2 d x^2}}-\frac{4 x^2 (a+b \operatorname{ArcCosh}[c x])}{3 c^4 d^2 \sqrt{d-c^2 d x^2}}+\frac{x^4 (a+b \operatorname{ArcCosh}[c x])}{3 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2}}-\frac{8 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{3 c^6 d^2 \sqrt{d-c^2 d x^2}}-\frac{11 b \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcTanh}[c x]}{6 c^6 d^2 \sqrt{d-c^2 d x^2}}$$

Problem 126: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a+b \operatorname{ArcCosh}[c x])}{(d-c^2 d x^2)^{5/2}} \, dx$$

Optimal (type 3, 158 leaves, 4 steps):

$$\frac{b\,x\,\sqrt{d-c^2\,d\,x^2}}{6\,c^3\,d^3\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}}+\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,c^4\,d\,(d-c^2\,d\,x^2)^{3/2}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{c^4\,d^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{5\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}[c\,x]}{6\,c^4\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 243 leaves, 5 steps):

$$\frac{b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c^3\,d^2\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{c^4\,d^2\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c\,d^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}+\frac{(1-c\,x)^2\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^4\,d^2\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}-\frac{5\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTanh}[c\,x]}{6\,c^4\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 127: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\,(a+b\,\text{ArcCosh}[c\,x])}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c^3\,d\,(d-c^2\,d\,x^2)^{3/2}}+\frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])}{3\,d\,(d-c^2\,d\,x^2)^{3/2}}+\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[1-c^2\,x^2]}{6\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 160 leaves, 5 steps):

$$\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c^3\,d^2\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}+\frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])}{3\,d^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[1-c^2\,x^2]}{6\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 128: Result valid but suboptimal antiderivative.

$$\int\frac{x\,(a+b\,\text{ArcCosh}[c\,x])}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$\frac{b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c\,d\,(d-c^2\,d\,x^2)^{3/2}}+\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,c^2\,d\,(d-c^2\,d\,x^2)^{3/2}}+\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTanh}[c\,x]}{6\,c^2\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 154 leaves, 4 steps):

$$\frac{b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c\,d^2\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}+\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,c^2\,d^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTanh}[c\,x]}{6\,c^2\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 131: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcCosh}[c\,x]}{x^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 248 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{6\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)}-\frac{a+b\,\text{ArcCosh}[c\,x]}{d\,x\,\left(d-c^2\,d\,x^2\right)^{3/2}}+\frac{4\,c^2\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}}+$$
$$\frac{8\,c^2\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5\,b\,c\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}\left[1-c^2\,x^2\right]}{6\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 279 leaves, 6 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,d^2\,\left(1-c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}}+\frac{8\,c^2\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{d^2\,x\,\left(1-c\,x\right)\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{4\,c^2\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,\left(1-c\,x\right)\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{d^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{5\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{6\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcCosh}[c\,x]}{x^4\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 338 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{6\,d^3\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^3\,\sqrt{d-c^2\,d\,x^2}}{6\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)}-\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,d\,x^3\,\left(d-c^2\,d\,x^2\right)^{3/2}}-\frac{2\,c^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d\,x\,\left(d-c^2\,d\,x^2\right)^{3/2}}+$$
$$\frac{8\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}}+\frac{16\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{8\,b\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{3\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}\left[1-c^2\,x^2\right]}{3\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 383 leaves, 6 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,d^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,d^2\,\left(1-c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}}+\frac{16\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,d^2\,x^3\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,c^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d^2\,x\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{8\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}-\frac{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 134: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]}{\left(c-a^2\,c\,x^2\right)^{7/2}}\,dx$$

Optimal (type 3, 246 leaves, 8 steps):

$$\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{20\,a\,c^3\,\left(1-a^2\,x^2\right)^2\,\sqrt{c-a^2\,c\,x^2}}+\frac{2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{15\,a\,c^3\,\left(1-a^2\,x^2\right)\,\sqrt{c-a^2\,c\,x^2}}+$$
$$\frac{x\,\text{ArcCosh}[a\,x]}{5\,c\,\left(c-a^2\,c\,x^2\right)^{5/2}}+\frac{4\,x\,\text{ArcCosh}[a\,x]}{15\,c^2\,\left(c-a^2\,c\,x^2\right)^{3/2}}+\frac{8\,x\,\text{ArcCosh}[a\,x]}{15\,c^3\,\sqrt{c-a^2\,c\,x^2}}-\frac{4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Log}\left[1-a^2\,x^2\right]}{15\,a\,c^3\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 3, 276 leaves, 7 steps):

$$\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{20\,a\,c^3\,\left(1-a^2\,x^2\right)^2\,\sqrt{c-a^2\,c\,x^2}}+\frac{2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{15\,a\,c^3\,\left(1-a^2\,x^2\right)\,\sqrt{c-a^2\,c\,x^2}}+\frac{8\,x\,\text{ArcCosh}[a\,x]}{15\,c^3\,\sqrt{c-a^2\,c\,x^2}}+$$
$$\frac{x\,\text{ArcCosh}[a\,x]}{5\,c^3\,(1-a\,x)^2\,(1+a\,x)^2\,\sqrt{c-a^2\,c\,x^2}}+\frac{4\,x\,\text{ArcCosh}[a\,x]}{15\,c^3\,(1-a\,x)\,(1+a\,x)\,\sqrt{c-a^2\,c\,x^2}}-\frac{4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Log}\left[1-a^2\,x^2\right]}{15\,a\,c^3\,\sqrt{c-a^2\,c\,x^2}}$$

Problem 135: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\,\text{ArcCosh}[a\,x]}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$-\frac{3\,x^2\,\sqrt{-1+a\,x}}{16\,a^3\,\sqrt{1-a\,x}}-\frac{x^4\,\sqrt{-1+a\,x}}{16\,a\,\sqrt{1-a\,x}}-\frac{3\,x\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]}{8\,a^4}-\frac{x^3\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]}{4\,a^2}+\frac{3\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{16\,a^5\,\sqrt{1-a\,x}}$$

Result (type 3, 206 leaves, 6 steps):

$$-\frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}}{16a^3\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}}-\frac{3x(1-ax)(1+ax)\,\text{ArcCosh}[ax]}{8a^4\sqrt{1-a^2x^2}}-\frac{x^3(1-ax)(1+ax)\,\text{ArcCosh}[ax]}{4a^2\sqrt{1-a^2x^2}}+\frac{3\sqrt{-1+ax}\sqrt{1+ax}\,\text{ArcCosh}[ax]^2}{16a^5\sqrt{1-a^2x^2}}$$

Problem 136: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,\text{ArcCosh}[ax]}{\sqrt{1-a^2x^2}}\,dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$-\frac{2x\sqrt{-1+ax}}{3a^3\sqrt{1-ax}}-\frac{x^3\sqrt{-1+ax}}{9a\sqrt{1-ax}}-\frac{2\sqrt{1-a^2x^2}\,\text{ArcCosh}[ax]}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\,\text{ArcCosh}[ax]}{3a^2}$$

Result (type 3, 158 leaves, 5 steps):

$$-\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a^3\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}}-\frac{2(1-ax)(1+ax)\,\text{ArcCosh}[ax]}{3a^4\sqrt{1-a^2x^2}}-\frac{x^2(1-ax)(1+ax)\,\text{ArcCosh}[ax]}{3a^2\sqrt{1-a^2x^2}}$$

Problem 137: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\,\text{ArcCosh}[ax]}{\sqrt{1-a^2x^2}}\,dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$-\frac{x^2\sqrt{-1+ax}}{4a\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\,\text{ArcCosh}[ax]}{2a^2}+\frac{\sqrt{-1+ax}\,\text{ArcCosh}[ax]^2}{4a^3\sqrt{1-ax}}$$

Result (type 3, 125 leaves, 4 steps):

$$-\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{4a\sqrt{1-a^2x^2}}-\frac{x(1-ax)(1+ax)\,\text{ArcCosh}[ax]}{2a^2\sqrt{1-a^2x^2}}+\frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{ArcCosh}[ax]^2}{4a^3\sqrt{1-a^2x^2}}$$

Problem 138: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\text{ArcCosh}[ax]}{\sqrt{1-a^2x^2}}\,dx$$

Optimal (type 3, 49 leaves, 2 steps):

$$-\frac{x\sqrt{-1+a x}}{a\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]}{a^2}$$

Result (type 3, 73 leaves, 3 steps):

$$-\frac{x\sqrt{-1+a x}\ \sqrt{1+a x}}{a\sqrt{1-a^2 x^2}}-\frac{(1-a x)\ (1+a x)\ \text{ArcCosh}[a x]}{a^2\sqrt{1-a^2 x^2}}$$

Problem 139: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2}{2 a\sqrt{1-a x}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2}{2 a\sqrt{1-a^2 x^2}}$$

Problem 140: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]}{x\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 4, 103 leaves, 6 steps):

$$\frac{2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{i\sqrt{-1+a x}\ \text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{i\sqrt{-1+a x}\ \text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}$$

Result (type 4, 142 leaves, 7 steps):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$

$$\frac{i\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{i\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 141: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$-\frac{\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}{x}-\frac{a\,\sqrt{-1+a\,x}\;\text{Log}[x]}{\sqrt{1-a\,x}}$$

Result (type 3, 72 leaves, 3 steps):

$$-\frac{(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{x\,\sqrt{1-a^2\,x^2}}-\frac{a\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{Log}[x]}{\sqrt{1-a^2\,x^2}}$$

Problem 142: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 4, 167 leaves, 8 steps):

$$\frac{a\,\sqrt{-1+a\,x}}{2\,x\,\sqrt{1-a\,x}}-\frac{\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}{2\,x^2}+\frac{a^2\,\sqrt{-1+a\,x}\;\text{ArcCosh}[a\,x]\;\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{i\,a^2\,\sqrt{-1+a\,x}\;\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a\,x}}+\frac{i\,a^2\,\sqrt{-1+a\,x}\;\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a\,x}}$$

Result (type 4, 230 leaves, 9 steps):

$$\frac{a\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}}{2\,x\,\sqrt{1-a^2\,x^2}}-\frac{(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{2\,x^2\,\sqrt{1-a^2\,x^2}}+\frac{a^2\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{ArcCosh}[a\,x]\;\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-\frac{i\,a^2\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}+\frac{i\,a^2\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}$$

Problem 143: Result valid but suboptimal antiderivative.

$$\int \frac{(f\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 5, 98 leaves, 1 step):

$$\frac{2\,(f\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{5\,f}+\frac{4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{35\,f^2\,\sqrt{1-c\,x}}$$

Result (type 5, 111 leaves, 2 steps):

$$\frac{2\,(f\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{5\,f}+\frac{4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{35\,f^2\,\sqrt{1-c^2\,x^2}}$$

Problem 144: Result optimal but 1 more steps used.

$$\int\frac{(f\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 5, 141 leaves, 1 step):

$$\frac{2\,(f\,x)^{5/2}\,\sqrt{1-c^2\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{5\,f\,\sqrt{d-c^2\,d\,x^2}}+\frac{4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{35\,f^2\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 5, 141 leaves, 2 steps):

$$\frac{2\,(f\,x)^{5/2}\,\sqrt{1-c^2\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{5\,f\,\sqrt{d-c^2\,d\,x^2}}+\frac{4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{35\,f^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 153: Result valid but suboptimal antiderivative.

$$\int(f\,x)^m\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,dx$$

Optimal (type 5, 278 leaves, 3 steps):

$$-\frac{b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}}{f^2\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{(f\,x)^{1+m}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{f\,(2+m)}+$$
$$\frac{(f\,x)^{1+m}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]}{f\,\left(2+3\,m+m^2\right)\,\sqrt{1-c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]}{f^2\,(1+m)\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 5, 288 leaves, 4 steps):

$$-\frac{b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}}{f^2\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{(f\,x)^{1+m}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{f\,(2+m)}+$$
$$\frac{(f\,x)^{1+m}\,\sqrt{1-c^2\,x^2}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]}{f\,\left(2+3\,m+m^2\right)\,(1-c\,x)\,(1+c\,x)}-$$
$$\frac{b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]}{f^2\,(1+m)\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 154: Result optimal but 1 more steps used.

$$\int\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 5, 176 leaves, 1 step):

$$\frac{(f\,x)^{1+m}\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]}{f\,(1+m)\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{b\,c\,(f\,x)^{2+m}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]}{f^2\,(1+m)\,(2+m)\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 5, 176 leaves, 2 steps):

$$\frac{(f\,x)^{1+m}\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]}{f\,(1+m)\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{b\,c\,(f\,x)^{2+m}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]}{f^2\,(1+m)\,(2+m)\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 160: Result optimal but 1 more steps used.

$$\int\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}}\,dx$$

Optimal (type 5, 188 leaves, 1 step):

$$\frac{(f\,x)^{1+m}\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]}{f\,(1+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}}+$$
$$\frac{b\,c\,(f\,x)^{2+m}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]}{f^2\,(1+m)\,(2+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}}$$

Result (type 5, 188 leaves, 2 steps):

$$\frac{(f\,x)^{1+m}\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]}{f\,(1+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}}+$$
$$\frac{b\,c\,(f\,x)^{2+m}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]}{f^2\,(1+m)\,(2+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}}$$

Problem 163: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,\text{ArcCosh}[a\,x]}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 5, 128 leaves, 1 step):

$$\frac{(f\,x)^{1+m}\,\text{ArcCosh}[a\,x]\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2\,x^2\right]}{f\,(1+m)}+\frac{a\,(f\,x)^{2+m}\sqrt{-1+a\,x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},a^2\,x^2\right]}{f^2\,(1+m)\,(2+m)\,\sqrt{1-a\,x}}$$

Result (type 5, 141 leaves, 2 steps):

$$\frac{(f\,x)^{1+m}\,\text{ArcCosh}[a\,x]\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2\,x^2\right]}{f\,(1+m)}+$$

$$\frac{a\,(f\,x)^{2+m}\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},a^2\,x^2\right]}{f^2\,(1+m)\,(2+m)\,\sqrt{1-a^2\,x^2}}$$

Problem 170: Result optimal but 1 more steps used.

$$\int x^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,dx$$

Optimal (type 3, 371 leaves, 16 steps):

$$-\frac{856\,b^2\,\sqrt{d-c^2\,d\,x^2}}{3375\,c^4}+\frac{22\,b^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}{3375\,c^2}+\frac{2}{125}\,b^2\,x^4\,\sqrt{d-c^2\,d\,x^2}+\frac{4\,a\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{4\,b^2\,x\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2\,b\,x^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{45\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2\,b\,c\,x^5\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{25\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{15\,c^4}-\frac{x^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{15\,c^2}+\frac{1}{5}\,x^4\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2$$

Result (type 3, 371 leaves, 17 steps):

$$-\frac{856\,b^2\,\sqrt{d-c^2\,d\,x^2}}{3375\,c^4}+\frac{22\,b^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}{3375\,c^2}+\frac{2}{125}\,b^2\,x^4\,\sqrt{d-c^2\,d\,x^2}+\frac{4\,a\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{4\,b^2\,x\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2\,b\,x^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{45\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2\,b\,c\,x^5\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{25\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{15\,c^4}-\frac{x^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{15\,c^2}+\frac{1}{5}\,x^4\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2$$

Problem 171: Result optimal but 1 more steps used.

$$\int x^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,dx$$

Optimal (type 3, 319 leaves, 11 steps):

$$-\frac{b^2 x \sqrt{d-c^2 d x^2}}{64 c^2}+\frac{1}{32} b^2 x^3 \sqrt{d-c^2 d x^2}-\frac{b^2 \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{64 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b x^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{8 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c x^4 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{x \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{8 c^2}+\frac{1}{4} x^3 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2-\frac{\sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{24 b c^3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 319 leaves, 12 steps):

$$-\frac{b^2 x \sqrt{d-c^2 d x^2}}{64 c^2}+\frac{1}{32} b^2 x^3 \sqrt{d-c^2 d x^2}-\frac{b^2 \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{64 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b x^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{8 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c x^4 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{x \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{8 c^2}+\frac{1}{4} x^3 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2-\frac{\sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{24 b c^3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 173: Result optimal but 1 more steps used.

$$\int \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2 \, dx$$

Optimal (type 3, 204 leaves, 5 steps):

$$\frac{1}{4} b^2 x \sqrt{d-c^2 d x^2}+\frac{b^2 \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{4 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c x^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{1}{2} x \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2-\frac{\sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{6 b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 204 leaves, 6 steps):

$$\frac{1}{4} b^2 x \sqrt{d-c^2 d x^2}+\frac{b^2 \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{4 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c x^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{1}{2} x \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2-\frac{\sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{6 b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 174: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{x}\,dx$$

Optimal (type 4, 402 leaves, 12 steps):

$$2 b^2 \sqrt{d-c^2 d x^2}-\frac{2 a b c x \sqrt{d-c^2 d x^2}}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b^2 c x \sqrt{d-c^2 d x^2}\,\text{ArcCosh}[c x]}{\sqrt{-1+c x} \sqrt{1+c x}}+\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2-\frac{2 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b^2 \sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b^2 \sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 402 leaves, 13 steps):

$$2 b^2 \sqrt{d-c^2 d x^2}-\frac{2 a b c x \sqrt{d-c^2 d x^2}}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b^2 c x \sqrt{d-c^2 d x^2}\,\text{ArcCosh}[c x]}{\sqrt{-1+c x} \sqrt{1+c x}}+\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2-\frac{2 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b^2 \sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b^2 \sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{x^2}\,dx$$

Optimal (type 4, 234 leaves, 7 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^2}{x}+\frac{c\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{c\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^3}{3\,b\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{2\,b\,c\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])\ \text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{b^2 c\sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Result (type 4, 234 leaves, 8 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^2}{x}-\frac{c\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{c\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^3}{3\,b\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{2\,b\,c\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{b^2 c\sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 176: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^2}{x^3}\,dx$$

Optimal (type 4, 427 leaves, 12 steps):

$$-\frac{b\,c\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])}{x\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^2}{2\,x^2}+$$
$$\frac{c^2\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^2\ \text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{b^2 c^2\sqrt{d-c^2 d x^2}\ \text{ArcTan}\left[\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{i\,b\,c^2\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])\ \text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{i\,b\,c^2\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])\ \text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{i\,b^2 c^2\sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{i\,b^2 c^2\sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Result (type 4, 427 leaves, 13 steps):

$$-\frac{b\,c\,\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{x\,\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2}{2 x^2}+$$
$$\frac{c^2\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{b^2 c^2\sqrt{d-c^2 d x^2}\,\text{ArcTan}\left[\sqrt{-1+c x}\,\sqrt{1+c x}\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}-$$
$$\frac{i\,b\,c^2\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{i\,b\,c^2\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}+$$
$$\frac{i\,b^2 c^2\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{i\,b^2 c^2\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 177: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2}{x^4}\,dx$$

Optimal (type 4, 336 leaves, 11 steps):

$$\frac{b^2 c^2\sqrt{d-c^2 d x^2}}{3 x}-\frac{b^2 c^3\sqrt{d-c^2 d x^2}\,\text{ArcCosh}[c x]}{3\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{b\,c\,(1-c^2 x^2)\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{3 x^2\sqrt{-1+c x}\,\sqrt{1+c x}}-$$
$$\frac{c^3\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2}{3\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{(d-c^2 d x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c x])^2}{3 d x^3}-$$
$$\frac{2 b c^3\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c x]}\right]}{3\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{b^2 c^3\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c x]}\right]}{3\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Result (type 4, 344 leaves, 11 steps):

$$\frac{b^2 c^2\sqrt{d-c^2 d x^2}}{3 x}-\frac{b^2 c^3\sqrt{d-c^2 d x^2}\,\text{ArcCosh}[c x]}{3\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{b\,c\,(1-c^2 x^2)\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{3 x^2\sqrt{-1+c x}\,\sqrt{1+c x}}+$$
$$\frac{c^3\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2}{3\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{(1-c x)\,(1+c x)\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^2}{3 x^3}-$$
$$\frac{2 b c^3\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[c x]}\right]}{3\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{b^2 c^3\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c x]}\right]}{3\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 183: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c\,x])^2}{x^2}\,dx$$

Optimal (type 4, 453 leaves, 15 steps):

$$-\frac{1}{4} b^2 c^2 d\,x \sqrt{d-c^2 d x^2}-\frac{5 b^2 c\,d \sqrt{d-c^2 d x^2}\,\text{ArcCosh}[c\,x]}{4 \sqrt{-1+c\,x} \sqrt{1+c\,x}}+\frac{3 b\,c^3 d\,x^2 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2 \sqrt{-1+c\,x} \sqrt{1+c\,x}}+$$
$$\frac{b\,c\,d\,\left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}-\frac{3}{2} c^2 d\,x \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2+$$
$$\frac{c\,d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}-\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c\,x])^2}{x}+\frac{c\,d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^3}{2 b \sqrt{-1+c\,x} \sqrt{1+c\,x}}+$$
$$\frac{2 b\,c\,d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}-\frac{b^2 c\,d \sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}$$

Result (type 4, 465 leaves, 15 steps):

$$-\frac{1}{4} b^2 c^2 d\,x \sqrt{d-c^2 d x^2}-\frac{5 b^2 c\,d \sqrt{d-c^2 d x^2}\,\text{ArcCosh}[c\,x]}{4 \sqrt{-1+c\,x} \sqrt{1+c\,x}}+\frac{3 b\,c^3 d\,x^2 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2 \sqrt{-1+c\,x} \sqrt{1+c\,x}}+$$
$$\frac{b\,c\,d\,\left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}-\frac{3}{2} c^2 d\,x \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2-\frac{c\,d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}-$$
$$\frac{d\,(1-c\,x)\,(1+c\,x) \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{x}+\frac{c\,d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^3}{2 b \sqrt{-1+c\,x} \sqrt{1+c\,x}}+$$
$$\frac{2 b\,c\,d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}+\frac{b^2 c\,d \sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x} \sqrt{1+c\,x}}$$

Problem 185: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c\,x])^2}{x^4}\,dx$$

Optimal (type 4, 426 leaves, 18 steps):

$$\frac{b^2 c^2 d \sqrt{d-c^2 d x^2}}{3 x}-\frac{b^2 c^3 d \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{3 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{3 x^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{c^2 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{x}-$$
$$\frac{4 c^3 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{3 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])^2}{3 x^3}-\frac{c^3 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^3}{3 b \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{8 b c^3 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\ \text{Log}\left[1+e^{-2\,\text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{4 b^2 c^3 d \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 438 leaves, 18 steps):

$$\frac{b^2 c^2 d \sqrt{d-c^2 d x^2}}{3 x}-\frac{b^2 c^3 d \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{3 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{3 x^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{c^2 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{x}+$$
$$\frac{4 c^3 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{3 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{d (1-c x) (1+c x) \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{3 x^3}-\frac{c^3 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^3}{3 b \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{8 b c^3 d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{4 b^2 c^3 d \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 191: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} (a+b\,\text{ArcCosh}[c x])^2}{x^2}\, dx$$

Optimal (type 4, 607 leaves, 25 steps):

$$-\frac{31}{64} b^2 c^2 d^2 x \sqrt{d-c^2 d x^2}-\frac{1}{32} b^2 c^2 d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2}-\frac{89 b^2 c d^2 \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{64 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{15 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{8 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c d^2 \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{\sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{b c d^2 \left(1-c^2 x^2\right)^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2+\frac{c d^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{\sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{5}{4} c^2 d x \left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])^2-\frac{\left(d-c^2 d x^2\right)^{5/2} (a+b\,\text{ArcCosh}[c x])^2}{x}+\frac{5 c d^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^3}{8 b \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{2 b c d^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\ \text{Log}\left[1+e^{-2\,\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{b^2 c d^2 \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 638 leaves, 24 steps):

$$-\frac{31}{64} b^2 c^2 d^2 x \sqrt{d-c^2 d x^2} - \frac{1}{32} b^2 c^2 d^2 x (1-c x)(1+c x)\sqrt{d-c^2 d x^2} - \frac{89 b^2 c d^2 \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{64 \sqrt{-1+c x}\, \sqrt{1+c x}} +$$
$$\frac{15 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{8 \sqrt{-1+c x}\, \sqrt{1+c x}} + \frac{b c d^2 (1-c^2 x^2) \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{\sqrt{-1+c x}\, \sqrt{1+c x}} -$$
$$\frac{b c d^2 (1-c^2 x^2)^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{8 \sqrt{-1+c x}\, \sqrt{1+c x}} - \frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2 -$$
$$\frac{c d^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{\sqrt{-1+c x}\, \sqrt{1+c x}} - \frac{5}{4} c^2 d^2 x (1-c x)(1+c x)\sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2 -$$
$$\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{x} + \frac{5 c d^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{8 b \sqrt{-1+c x}\, \sqrt{1+c x}} +$$
$$\frac{2 b c d^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])\, \text{Log}\left[1+e^{2 \text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\, \sqrt{1+c x}} + \frac{b^2 c d^2 \sqrt{d-c^2 d x^2}\, \text{PolyLog}\left[2, -e^{2 \text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\, \sqrt{1+c x}}$$

Problem 193: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{5/2} (a+b\, \text{ArcCosh}[c x])^2}{x^4}\, dx$$

Optimal (type 4, 638 leaves, 30 steps):

$$\frac{7}{12} b^2 c^4 d^2 x \sqrt{d-c^2 d x^2} + \frac{b^2 c^2 d^2 (1-c x)(1+c x)\sqrt{d-c^2 d x^2}}{3 x} + \frac{23 b^2 c^3 d^2 \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{12 \sqrt{-1+c x}\, \sqrt{1+c x}} -$$
$$\frac{5 b c^5 d^2 x^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{2 \sqrt{-1+c x}\, \sqrt{1+c x}} - \frac{7 b c^3 d^2 (1-c^2 x^2) \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{3 \sqrt{-1+c x}\, \sqrt{1+c x}} -$$
$$\frac{b c d^2 (1-c^2 x^2)^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{3 x^2 \sqrt{-1+c x}\, \sqrt{1+c x}} + \frac{5}{2} c^4 d^2 x \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2 - \frac{7 c^3 d^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{3 \sqrt{-1+c x}\, \sqrt{1+c x}} +$$
$$\frac{5 c^2 d (d-c^2 d x^2)^{3/2} (a+b\, \text{ArcCosh}[c x])^2}{3 x} - \frac{(d-c^2 d x^2)^{5/2} (a+b\, \text{ArcCosh}[c x])^2}{3 x^3} - \frac{5 c^3 d^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{6 b \sqrt{-1+c x}\, \sqrt{1+c x}} -$$
$$\frac{14 b c^3 d^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])\, \text{Log}\left[1+e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x}\, \sqrt{1+c x}} + \frac{7 b^2 c^3 d^2 \sqrt{d-c^2 d x^2}\, \text{PolyLog}\left[2, -e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x}\, \sqrt{1+c x}}$$

Result (type 4, 669 leaves, 29 steps):

$$\frac{7}{12} b^2 c^4 d^2 x \sqrt{d-c^2 d x^2} + \frac{b^2 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2}}{3 x} + \frac{23 b^2 c^3 d^2 \sqrt{d-c^2 d x^2} \text{ArcCosh}[c x]}{12 \sqrt{-1+c x} \sqrt{1+c x}} -$$
$$\frac{5 b c^5 d^2 x^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{2 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{7 b c^3 d^2 (1-c^2 x^2) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{3 \sqrt{-1+c x} \sqrt{1+c x}} -$$
$$\frac{b c d^2 (1-c^2 x^2)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{3 x^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{5}{2} c^4 d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2 +$$
$$\frac{7 c^3 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{5 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 x} -$$
$$\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 x^3} - \frac{5 c^3 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^3}{6 b \sqrt{-1+c x} \sqrt{1+c x}} -$$
$$\frac{14 b c^3 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1+e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{7 b^2 c^3 d^2 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2, -e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 194: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 421 leaves, 16 steps):

$$-\frac{16 a b x \sqrt{-1+c x} \sqrt{1+c x}}{15 c^5 \sqrt{d-c^2 d x^2}} - \frac{4144 b^2 (1-c x) (1+c x)}{3375 c^6 \sqrt{d-c^2 d x^2}} - \frac{272 b^2 x^2 (1-c x) (1+c x)}{3375 c^4 \sqrt{d-c^2 d x^2}} - \frac{2 b^2 x^4 (1-c x) (1+c x)}{125 c^2 \sqrt{d-c^2 d x^2}} -$$
$$\frac{16 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{15 c^5 \sqrt{d-c^2 d x^2}} - \frac{8 b x^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{45 c^3 \sqrt{d-c^2 d x^2}} - \frac{2 b x^5 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{25 c \sqrt{d-c^2 d x^2}} -$$
$$\frac{8 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^6 d} - \frac{4 x^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^4 d} - \frac{x^4 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{5 c^2 d}$$

Result (type 3, 445 leaves, 17 steps):

$$-\frac{16\,a\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{15\,c^5\,\sqrt{d-c^2\,d\,x^2}}-\frac{4144\,b^2\,(1-c\,x)\,(1+c\,x)}{3375\,c^6\,\sqrt{d-c^2\,d\,x^2}}-\frac{272\,b^2\,x^2\,(1-c\,x)\,(1+c\,x)}{3375\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b^2\,x^4\,(1-c\,x)\,(1+c\,x)}{125\,c^2\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{16\,b^2\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{15\,c^5\,\sqrt{d-c^2\,d\,x^2}}-\frac{8\,b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{45\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b\,x^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{25\,c\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{8\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{15\,c^6\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,x^2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{15\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{x^4\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{5\,c^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 195: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 355 leaves, 11 steps):

$$-\frac{15\,b^2\,x\,(1-c\,x)\,(1+c\,x)}{64\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{b^2\,x^3\,(1-c\,x)\,(1+c\,x)}{32\,c^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{15\,b^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{64\,c^5\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3\,b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{8\,c^4\,d}-\frac{x^3\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,c^2\,d}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^3}{8\,b\,c^5\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 371 leaves, 12 steps):

$$-\frac{15\,b^2\,x\,(1-c\,x)\,(1+c\,x)}{64\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{b^2\,x^3\,(1-c\,x)\,(1+c\,x)}{32\,c^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{15\,b^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{64\,c^5\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3\,b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3\,x\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{x^3\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,c^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^3}{8\,b\,c^5\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 196: Result valid but suboptimal antiderivative.

$$\int \frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 292 leaves, 9 steps):

$$-\frac{4 a b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{40 b^2 (1-c x) (1+c x)}{27 c^4 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 x^2 (1-c x) (1+c x)}{27 c^2 \sqrt{d-c^2 d x^2}}-\frac{4 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{2 b x^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{9 c \sqrt{d-c^2 d x^2}}-\frac{2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 c^4 d}-\frac{x^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 c^2 d}$$

Result (type 3, 308 leaves, 10 steps):

$$-\frac{4 a b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{40 b^2 (1-c x) (1+c x)}{27 c^4 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 x^2 (1-c x) (1+c x)}{27 c^2 \sqrt{d-c^2 d x^2}}-\frac{4 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{2 b x^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{9 c \sqrt{d-c^2 d x^2}}-\frac{2 (1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{3 c^4 \sqrt{d-c^2 d x^2}}-\frac{x^2 (1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{3 c^2 \sqrt{d-c^2 d x^2}}$$

Problem 197: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 226 leaves, 5 steps):

$$-\frac{b^2 x (1-c x) (1+c x)}{4 c^2 \sqrt{d-c^2 d x^2}}+\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{4 c^3 \sqrt{d-c^2 d x^2}}-\frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{2 c \sqrt{d-c^2 d x^2}}-\frac{x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{2 c^2 d}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^3}{6 b c^3 \sqrt{d-c^2 d x^2}}$$

Result (type 3, 234 leaves, 6 steps):

$$-\frac{b^2 x (1-c x) (1+c x)}{4 c^2 \sqrt{d-c^2 d x^2}}+\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{4 c^3 \sqrt{d-c^2 d x^2}}-\frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{2 c \sqrt{d-c^2 d x^2}}-\frac{x (1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{2 c^2 \sqrt{d-c^2 d x^2}}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^3}{6 b c^3 \sqrt{d-c^2 d x^2}}$$

Problem 198: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$-\frac{2\,a\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{c\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b^2\,(1-c\,x)\,(1+c\,x)}{c^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b^2\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{c\,\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{c^2\,d}$$

Result (type 3, 163 leaves, 5 steps):

$$-\frac{2\,a\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{c\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b^2\,(1-c\,x)\,(1+c\,x)}{c^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b^2\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{c\,\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{c^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 199: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^3}{3\,b\,c\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 53 leaves, 2 steps):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^3}{3\,b\,c\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 200: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcCosh}[c\,x])^2}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 273 leaves, 8 steps):

$$\frac{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{2\,i\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{2\,i\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{2\,i\,b^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{2\,i\,b^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 273 leaves, 9 steps):

$$\frac{2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{2\,i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+$$

$$\frac{2\,i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+$$

$$\frac{2\,i\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{2\,i\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 201: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x^2\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 186 leaves, 6 steps):

$$-\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{d\,x}-$$

$$\frac{2\,b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{b^2\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,-e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 194 leaves, 7 steps):

$$\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x\sqrt{d-c^2\,d\,x^2}}-$$

$$\frac{2\,b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{b^2\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 202: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x^3\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 430 leaves, 12 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x\,\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,d\,x^2}+\frac{c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 438 leaves, 13 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x\,\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,x^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 203: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x^4\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 328 leaves, 9 steps):

$$\frac{b^2 c^2 (1-c x) (1+c x)}{3 x \sqrt{d-c^2 d x^2}}+\frac{b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{3 x^2 \sqrt{d-c^2 d x^2}}-\frac{2 c^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{3 \sqrt{d-c^2 d x^2}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 d x^3}-\frac{2 c^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 d x}-\frac{4 b c^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1+e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{d-c^2 d x^2}}+\frac{2 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x} \text{PolyLog}\left[2,-e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{d-c^2 d x^2}}$$

Result (type 4, 344 leaves, 10 steps):

$$\frac{b^2 c^2 (1-c x) (1+c x)}{3 x \sqrt{d-c^2 d x^2}}+\frac{b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{3 x^2 \sqrt{d-c^2 d x^2}}+\frac{2 c^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{3 \sqrt{d-c^2 d x^2}}-\frac{(1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{3 x^3 \sqrt{d-c^2 d x^2}}-\frac{2 c^2 (1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{3 x \sqrt{d-c^2 d x^2}}-\frac{4 b c^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1+e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x} \text{PolyLog}\left[2,-e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{d-c^2 d x^2}}$$

Problem 209: Result optimal but 1 more steps used.

$$\int \frac{(a+b \text{ArcCosh}[c x])^2}{\left(d-c^2 d x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 198 leaves, 6 steps):

$$\frac{x (a+b \text{ArcCosh}[c x])^2}{d \sqrt{d-c^2 d x^2}}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{c d \sqrt{d-c^2 d x^2}}-\frac{2 b \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1-e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}-\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \text{PolyLog}\left[2,e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}$$

Result (type 4, 198 leaves, 7 steps):

$$\frac{x (a+b \text{ArcCosh}[c x])^2}{d \sqrt{d-c^2 d x^2}}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{c d \sqrt{d-c^2 d x^2}}-\frac{2 b \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1-e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}-\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \text{PolyLog}\left[2,e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}$$

Problem 225: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \operatorname{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 243 leaves, 11 steps):

$$-\frac{15 x \sqrt{1-a x} \sqrt{1+a x}}{64 a^4}-\frac{x^3 \sqrt{1-a x} \sqrt{1+a x}}{32 a^2}+\frac{15 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{64 a^5 \sqrt{1-a x}}-\frac{3 x^2 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{8 a^3 \sqrt{1-a x}}-\frac{x^4 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{8 a \sqrt{1-a x}}-\frac{3 x \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}{8 a^4}-\frac{x^3 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}{4 a^2}+\frac{\sqrt{-1+a x} \operatorname{ArcCosh}[a x]^3}{8 a^5 \sqrt{1-a x}}$$

Result (type 3, 329 leaves, 12 steps):

$$-\frac{15 x (1-a x) (1+a x)}{64 a^4 \sqrt{1-a^2 x^2}}-\frac{x^3 (1-a x) (1+a x)}{32 a^2 \sqrt{1-a^2 x^2}}+\frac{15 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{64 a^5 \sqrt{1-a^2 x^2}}-\frac{3 x^2 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{8 a^3 \sqrt{1-a^2 x^2}}-\frac{x^4 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{8 a \sqrt{1-a^2 x^2}}-\frac{3 x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^2}{8 a^4 \sqrt{1-a^2 x^2}}-\frac{x^3 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^2}{4 a^2 \sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^3}{8 a^5 \sqrt{1-a^2 x^2}}$$

Problem 226: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \operatorname{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 177 leaves, 8 steps):

$$-\frac{40 \sqrt{1-a x} \sqrt{1+a x}}{27 a^4}-\frac{2 x^2 \sqrt{1-a x} \sqrt{1+a x}}{27 a^2}-\frac{4 x \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{3 a^3 \sqrt{1-a x}}-\frac{2 x^3 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{9 a \sqrt{1-a x}}-\frac{2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}{3 a^4}-\frac{x^2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}{3 a^2}$$

Result (type 3, 237 leaves, 9 steps):

$$-\frac{40 (1-a x) (1+a x)}{27 a^4 \sqrt{1-a^2 x^2}}-\frac{2 x^2 (1-a x) (1+a x)}{27 a^2 \sqrt{1-a^2 x^2}}-\frac{4 x \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{3 a^3 \sqrt{1-a^2 x^2}}-\frac{2 x^3 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{9 a \sqrt{1-a^2 x^2}}-\frac{2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^2}{3 a^4 \sqrt{1-a^2 x^2}}-\frac{x^2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^2}{3 a^2 \sqrt{1-a^2 x^2}}$$

Problem 227: Result valid but suboptimal antiderivative.

$$\int \frac{x^2\,\text{ArcCosh}[a\,x]^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$-\frac{x\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}}{4\,a^2}+\frac{\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{4\,a^3\,\sqrt{1-a\,x}}-\frac{x^2\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{2\,a\,\sqrt{1-a\,x}}-\frac{x\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}{2\,a^2}+\frac{\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^3}{6\,a^3\,\sqrt{1-a\,x}}$$

Result (type 3, 207 leaves, 6 steps):

$$-\frac{x\,(1-a\,x)\,(1+a\,x)}{4\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{4\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{x^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{2\,a\,\sqrt{1-a^2\,x^2}}-\frac{x\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^2}{2\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^3}{6\,a^3\,\sqrt{1-a^2\,x^2}}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int \frac{x\,\text{ArcCosh}[a\,x]^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 79 leaves, 3 steps):

$$-\frac{2\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}}{a^2}-\frac{2\,x\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{a\,\sqrt{1-a\,x}}-\frac{\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}{a^2}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{2\,(1-a\,x)\,(1+a\,x)}{a^2\,\sqrt{1-a^2\,x^2}}-\frac{2\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{a\,\sqrt{1-a^2\,x^2}}-\frac{(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^2}{a^2\,\sqrt{1-a^2\,x^2}}$$

Problem 229: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^3}{3\,a\,\sqrt{1-a\,x}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^3}{3\,a\,\sqrt{1-a^2\,x^2}}$$

Problem 230: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$\frac{2\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{2\,i\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{2\,i\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{2\,i\,\sqrt{-1+a\,x}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{2\,i\,\sqrt{-1+a\,x}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}$$

Result (type 4, 248 leaves, 9 steps):

$$\frac{2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-\frac{2\,i\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+\frac{2\,i\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+\frac{2\,i\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-\frac{2\,i\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}$$

Problem 231: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$\frac{a\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{\sqrt{1-a\,x}}-\frac{\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}{x}-\frac{2\,a\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{a\,\sqrt{-1+a\,x}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}$$

Result (type 4, 174 leaves, 7 steps):

$$\frac{a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}}-\frac{(1-a x)\ (1+a x)\ \text{ArcCosh}[a x]^2}{x\sqrt{1-a^2 x^2}}-\frac{2 a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-\frac{a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 232: Result valid but suboptimal antiderivative.

$$\int\frac{\text{ArcCosh}[a x]^2}{x^3\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 4, 296 leaves, 12 steps):

$$\frac{a\sqrt{-1+a x}\ \text{ArcCosh}[a x]}{x\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]^2}{2 x^2}+\frac{a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{a^2\sqrt{-1+a x}\ \text{ArcTan}\left[\sqrt{-1+a x}\ \sqrt{1+a x}\right]}{\sqrt{1-a x}}-\frac{i\,a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{i\,a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{i\,a^2\sqrt{-1+a x}\ \text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{i\,a^2\sqrt{-1+a x}\ \text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}$$

Result (type 4, 398 leaves, 13 steps):

$$\frac{a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]}{x\sqrt{1-a^2 x^2}}-\frac{(1-a x)\ (1+a x)\ \text{ArcCosh}[a x]^2}{2 x^2\sqrt{1-a^2 x^2}}+\frac{a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-\frac{a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcTan}\left[\sqrt{-1+a x}\ \sqrt{1+a x}\right]}{\sqrt{1-a^2 x^2}}-\frac{i\,a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{i\,a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{i\,a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-\frac{i\,a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 233: Result valid but suboptimal antiderivative.

$$\int (f x)^m\ \left(d-c^2 d x^2\right)^{5/2}\ (a+b\,\text{ArcCosh}[c x])^2\,dx$$

Optimal (type 8, 1153 leaves, 22 steps):

$$-\frac{10 b^2 c^2 d^2 (f x)^{3+m} \sqrt{d-c^2 d x^2}}{f^3 (4+m)^3 (6+m)}-\frac{2 b^2 c^2 d^2 (52+15 m+m^2) (f x)^{3+m} (1-c^2 x^2) \sqrt{d-c^2 d x^2}}{f^3 (4+m)^2 (6+m)^3 (1-c x) (1+c x)}+\frac{2 b^2 c^4 d^2 (f x)^{5+m} (1-c^2 x^2) \sqrt{d-c^2 d x^2}}{f^5 (6+m)^3 (1-c x) (1+c x)}-$$

$$\frac{2 b c d^2 (f x)^{2+m} \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{f^2 (2+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}-\frac{30 b c d^2 (f x)^{2+m} \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{f^2 (2+m)^2 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{10 b c d^2 (f x)^{2+m} \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{f^2 (2+m) (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}+\frac{10 b c^3 d^2 (f x)^{4+m} \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{f^4 (4+m)^2 (6+m) \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{4 b c^3 d^2 (f x)^{4+m} \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{f^4 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b c^5 d^2 (f x)^{6+m} \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{f^6 (6+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{15 d^2 (f x)^{1+m} \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{f (6+m) (8+6 m+m^2)}+\frac{5 d (f x)^{1+m} (d-c^2 d x^2)^{3/2} (a+b \text{ArcCosh}[c x])^2}{f (4+m) (6+m)}+$$

$$\frac{(f x)^{1+m} (d-c^2 d x^2)^{5/2} (a+b \text{ArcCosh}[c x])^2}{f (6+m)}-\frac{30 b^2 c^2 d^2 (f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2 x^2\right]}{f^3 (2+m)^2 (3+m) (4+m) (6+m) (1-c x) (1+c x)}-$$

$$\frac{10 b^2 c^2 d^2 (10+3 m) (f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2 x^2\right]}{f^3 (2+m) (3+m) (4+m)^3 (6+m) (1-c x) (1+c x)}-$$

$$\frac{2 b^2 c^2 d^2 (264+130 m+15 m^2) (f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2 x^2\right]}{f^3 (2+m) (3+m) (4+m)^2 (6+m)^3 (1-c x) (1+c x)}+$$

$$\frac{15 d^3 \text{Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}},x\right]}{(6+m) (8+6 m+m^2)}$$

Result (type 8, 73 leaves, 1 step):

$$\frac{d^2 \sqrt{d-c^2 d x^2} \text{Unintegrable}\left[(f x)^m (-1+c x)^{5/2} (1+c x)^{5/2} (a+b \text{ArcCosh}[c x])^2,x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 234: Result valid but suboptimal antiderivative.

$$\int (f x)^m (d-c^2 d x^2)^{3/2} (a+b \text{ArcCosh}[c x])^2 \, dx$$

Optimal (type 8, 583 leaves, 13 steps):

$$-\frac{2 b^2 c^2 d (f x)^{3+m} \sqrt{d-c^2 d x^2}}{f^3 (4+m)^3}-\frac{6 b c d (f x)^{2+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{f^2 (2+m)^2 (4+m) \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b c d (f x)^{2+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{f^2 (2+m) (4+m) \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{2 b c^3 d (f x)^{4+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{f^4 (4+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{3 d (f x)^{1+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{f (8+6 m+m^2)}+$$

$$\frac{(f x)^{1+m} (d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x])^2}{f (4+m)}-\frac{6 b^2 c^2 d (f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m)^2 (3+m) (4+m) (1-c x) (1+c x)}-$$

$$\frac{2 b^2 c^2 d (10+3 m) (f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m) (3+m) (4+m)^3 (1-c x) (1+c x)}+\frac{3 d^2 \text{Unintegrable}\left[\frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}}, x\right]}{8+6 m+m^2}$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{d \sqrt{d-c^2 d x^2} \text{Unintegrable}\left[(f x)^m (-1+c x)^{3/2} (1+c x)^{3/2} (a+b \operatorname{ArcCosh}[c x])^2, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 235: Result valid but suboptimal antiderivative.

$$\int (f x)^m \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2 \, dx$$

Optimal (type 8, 239 leaves, 5 steps):

$$-\frac{2 b c (f x)^{2+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{f^2 (2+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{(f x)^{1+m} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{f (2+m)}-$$

$$\frac{2 b^2 c^2 (f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m)^2 (3+m) (1-c x) (1+c x)}+\frac{d \text{Unintegrable}\left[\frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}}, x\right]}{2+m}$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{d-c^2 d x^2} \text{Unintegrable}\left[(f x)^m \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 236: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} \text{ Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{\sqrt{-1+c x} \sqrt{1+c x}}, x\right]}{\sqrt{d-c^2 d x^2}}$$

Problem 237: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{(d-c^2 d x^2)^{3/2}} \, dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{(d-c^2 d x^2)^{3/2}}, x\right]$$

Result (type 8, 74 leaves, 1 step):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x} \text{ Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{(-1+c x)^{3/2} (1+c x)^{3/2}}, x\right]}{d \sqrt{d-c^2 d x^2}}$$

Problem 238: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{(d-c^2 d x^2)^{5/2}} \, dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{(d-c^2 d x^2)^{5/2}}, x\right]$$

Result (type 8, 73 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} \text{ Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^2}{(-1+c x)^{5/2} (1+c x)^{5/2}}, x\right]}{d^2 \sqrt{d-c^2 d x^2}}$$

Problem 239: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m \operatorname{ArcCosh}[c x]^2}{\sqrt{1-c^2 x^2}} \, dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m \operatorname{ArcCosh}[c x]^2}{\sqrt{1-c^2 x^2}}, x\right]$$

Result (type 8, 65 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} \, \text{Unintegrable}\left[\frac{(f x)^m \operatorname{ArcCosh}[c x]^2}{\sqrt{-1+c x} \sqrt{1+c x}}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 248: Result optimal but 1 more steps used.

$$\int \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^3 \, dx$$

Optimal (type 3, 231 leaves, 6 steps):

$$-\frac{3 a x^2 \sqrt{c-a^2 c x^2}}{8 \sqrt{-1+a x} \sqrt{1+a x}}+\frac{3}{4} x \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]+\frac{3 \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^2}{8 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{3 a x^2 \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^2}{4 \sqrt{-1+a x} \sqrt{1+a x}}+\frac{1}{2} x \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^3-\frac{\sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^4}{8 a \sqrt{-1+a x} \sqrt{1+a x}}$$

Result (type 3, 231 leaves, 7 steps):

$$-\frac{3 a x^2 \sqrt{c-a^2 c x^2}}{8 \sqrt{-1+a x} \sqrt{1+a x}}+\frac{3}{4} x \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]+\frac{3 \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^2}{8 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{3 a x^2 \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^2}{4 \sqrt{-1+a x} \sqrt{1+a x}}+\frac{1}{2} x \sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^3-\frac{\sqrt{c-a^2 c x^2} \operatorname{ArcCosh}[a x]^4}{8 a \sqrt{-1+a x} \sqrt{1+a x}}$$

Problem 249: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcCosh}[a x]^3}{\sqrt{c-a^2 c x^2}} \, dx$$

Optimal (type 3, 46 leaves, 1 step):

$$\frac{\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]^4}{4 a \sqrt{c-a^2 c x^2}}$$

Result (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]^4}{4 a \sqrt{c-a^2 c x^2}}$$

Problem 250: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a x]^3}{\left(c-a^2 c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 241 leaves, 7 steps):

$$\frac{x\,\text{ArcCosh}[a x]^3}{c \sqrt{c-a^2 c x^2}}+\frac{\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]^3}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]^2\,\text{Log}\left[1-e^{2\,\text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]\,\text{PolyLog}\left[2, e^{2\,\text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}+\frac{3 \sqrt{-1+a x}\,\sqrt{1+a x}\,\text{PolyLog}\left[3, e^{2\,\text{ArcCosh}[a x]}\right]}{2 a c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 241 leaves, 8 steps):

$$\frac{x\,\text{ArcCosh}[a x]^3}{c \sqrt{c-a^2 c x^2}}+\frac{\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]^3}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]^2\,\text{Log}\left[1-e^{2\,\text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\,\sqrt{1+a x}\,\text{ArcCosh}[a x]\,\text{PolyLog}\left[2, e^{2\,\text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}+\frac{3 \sqrt{-1+a x}\,\sqrt{1+a x}\,\text{PolyLog}\left[3, e^{2\,\text{ArcCosh}[a x]}\right]}{2 a c \sqrt{c-a^2 c x^2}}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\,\text{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 315 leaves, 13 steps):

$$-\frac{45\,x^2\,\sqrt{-1+a\,x}}{128\,a^3\,\sqrt{1-a\,x}}-\frac{3\,x^4\,\sqrt{-1+a\,x}}{128\,a\,\sqrt{1-a\,x}}-\frac{45\,x\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{64\,a^4}-\frac{3\,x^3\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{32\,a^2}+\frac{45\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{128\,a^5\,\sqrt{1-a\,x}}-\frac{9\,x^2\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{16\,a^3\,\sqrt{1-a\,x}}-\frac{3\,x^4\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{16\,a\,\sqrt{1-a\,x}}-\frac{3\,x\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^3}{8\,a^4}-\frac{x^3\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^3}{4\,a^2}+\frac{3\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^4}{32\,a^5\,\sqrt{1-a\,x}}$$

Result (type 3, 427 leaves, 14 steps):

$$-\frac{45\,x^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{128\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{3\,x^4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{128\,a\,\sqrt{1-a^2\,x^2}}-\frac{45\,x\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{64\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{3\,x^3\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{32\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{45\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{128\,a^5\,\sqrt{1-a^2\,x^2}}-\frac{9\,x^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{16\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{3\,x^4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{16\,a\,\sqrt{1-a^2\,x^2}}-\frac{3\,x\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{8\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{x^3\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{4\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{3\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^4}{32\,a^5\,\sqrt{1-a^2\,x^2}}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,\text{ArcCosh}[a\,x]^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 243 leaves, 10 steps):

$$-\frac{40\,x\,\sqrt{-1+a\,x}}{9\,a^3\,\sqrt{1-a\,x}}-\frac{2\,x^3\,\sqrt{-1+a\,x}}{27\,a\,\sqrt{1-a\,x}}-\frac{40\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{9\,a^4}-\frac{2\,x^2\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{9\,a^2}-\frac{2\,x\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{a^3\,\sqrt{1-a\,x}}-\frac{x^3\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{3\,a\,\sqrt{1-a\,x}}-\frac{2\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^3}{3\,a^4}-\frac{x^2\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^3}{3\,a^2}$$

Result (type 3, 329 leaves, 11 steps):

$$-\frac{40\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{9\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{2\,x^3\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{27\,a\,\sqrt{1-a^2\,x^2}}-\frac{40\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{9\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{2\,x^2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{9\,a^2\,\sqrt{1-a^2\,x^2}}-\frac{2\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{a^3\,\sqrt{1-a^2\,x^2}}-\frac{x^3\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{3\,a\,\sqrt{1-a^2\,x^2}}-\frac{2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{3\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{x^2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{3\,a^2\,\sqrt{1-a^2\,x^2}}$$

Problem 255: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \text{ArcCosh}[a\,x]^3}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$-\frac{3 x^2 \sqrt{-1+a x}}{8 a \sqrt{1-a x}}-\frac{3 x \sqrt{1-a x} \sqrt{1+a x}\, \text{ArcCosh}[a x]}{4 a^2}+\frac{3 \sqrt{-1+a x}\, \text{ArcCosh}[a x]^2}{8 a^3 \sqrt{1-a x}}-\frac{3 x^2 \sqrt{-1+a x}\, \text{ArcCosh}[a x]^2}{4 a \sqrt{1-a x}}-\frac{x \sqrt{1-a^2 x^2}\, \text{ArcCosh}[a x]^3}{2 a^2}+\frac{\sqrt{-1+a x}\, \text{ArcCosh}[a x]^4}{8 a^3 \sqrt{1-a x}}$$

Result (type 3, 257 leaves, 7 steps):

$$-\frac{3 x^2 \sqrt{-1+a x} \sqrt{1+a x}}{8 a \sqrt{1-a^2 x^2}}-\frac{3 x (1-a x) (1+a x)\, \text{ArcCosh}[a x]}{4 a^2 \sqrt{1-a^2 x^2}}+\frac{3 \sqrt{-1+a x} \sqrt{1+a x}\, \text{ArcCosh}[a x]^2}{8 a^3 \sqrt{1-a^2 x^2}}-\frac{3 x^2 \sqrt{-1+a x} \sqrt{1+a x}\, \text{ArcCosh}[a x]^2}{4 a \sqrt{1-a^2 x^2}}-\frac{x (1-a x) (1+a x)\, \text{ArcCosh}[a x]^3}{2 a^2 \sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x} \sqrt{1+a x}\, \text{ArcCosh}[a x]^4}{8 a^3 \sqrt{1-a^2 x^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int \frac{x \,\text{ArcCosh}[a\,x]^3}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$-\frac{6 x \sqrt{-1+a x}}{a \sqrt{1-a x}}-\frac{6 \sqrt{1-a x} \sqrt{1+a x}\, \text{ArcCosh}[a x]}{a^2}-\frac{3 x \sqrt{-1+a x}\, \text{ArcCosh}[a x]^2}{a \sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\, \text{ArcCosh}[a x]^3}{a^2}$$

Result (type 3, 153 leaves, 5 steps):

$$-\frac{6 x \sqrt{-1+a x} \sqrt{1+a x}}{a \sqrt{1-a^2 x^2}}-\frac{6 (1-a x) (1+a x)\, \text{ArcCosh}[a x]}{a^2 \sqrt{1-a^2 x^2}}-\frac{3 x \sqrt{-1+a x} \sqrt{1+a x}\, \text{ArcCosh}[a x]^2}{a \sqrt{1-a^2 x^2}}-\frac{(1-a x) (1+a x)\, \text{ArcCosh}[a x]^3}{a^2 \sqrt{1-a^2 x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^3}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+a x}\ \text{ArcCosh}[a x]^4}{4 a \sqrt{1-a x}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^4}{4 a \sqrt{1-a^2 x^2}}$$

Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]^3}{x \sqrt{1-a^2 x^2}}\, dx$$

Optimal (type 4, 265 leaves, 10 steps):

$$\frac{2 \sqrt{-1+a x}\ \text{ArcCosh}[a x]^3 \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{3 i \sqrt{-1+a x}\ \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+$$
$$\frac{3 i \sqrt{-1+a x}\ \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{6 i \sqrt{-1+a x}\ \text{ArcCosh}[a x] \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-$$
$$\frac{6 i \sqrt{-1+a x}\ \text{ArcCosh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{6 i \sqrt{-1+a x}\ \text{PolyLog}\left[4,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{6 i \sqrt{-1+a x}\ \text{PolyLog}\left[4, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}$$

Result (type 4, 356 leaves, 11 steps):

$$\frac{2 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^3 \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$
$$\frac{3 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{3 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+$$
$$\frac{6 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x] \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-\frac{6 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$
$$\frac{6 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[4,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{6 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[4, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 259: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]^3}{x^2 \sqrt{1-a^2 x^2}}\, dx$$

Optimal (type 4, 166 leaves, 7 steps):

$$\frac{a\sqrt{-1+a x}\ \text{ArcCosh}[a x]^3}{\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]^3}{x}-\frac{3 a\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{3 a\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{3 a\sqrt{-1+a x}\ \text{PolyLog}\left[3,-e^{2\,\text{ArcCosh}[a x]}\right]}{2\sqrt{1-a x}}$$

Result (type 4, 229 leaves, 8 steps):

$$\frac{a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}}-\frac{(1-a x)\ (1+a x)\ \text{ArcCosh}[a x]^3}{x\sqrt{1-a^2 x^2}}-\frac{3 a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-\frac{3 a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{3 a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,-e^{2\,\text{ArcCosh}[a x]}\right]}{2\sqrt{1-a^2 x^2}}$$

Problem 260: Result valid but suboptimal antiderivative.

$$\int\frac{\text{ArcCosh}[a x]^3}{x^3\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 4, 460 leaves, 18 steps):

$$\frac{3 a\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2}{2 x\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]^3}{2 x^2}-\frac{6 a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]^3\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{3 i a^2\sqrt{-1+a x}\ \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{3 i a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2\ \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]}{2\sqrt{1-a x}}-\frac{3 i a^2\sqrt{-1+a x}\ \text{PolyLog}\left[2,i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{3 i a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2\ \text{PolyLog}\left[2,i e^{\text{ArcCosh}[a x]}\right]}{2\sqrt{1-a x}}+\frac{3 i a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{3 i a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[3,i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{3 i a^2\sqrt{-1+a x}\ \text{PolyLog}\left[4,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{3 i a^2\sqrt{-1+a x}\ \text{PolyLog}\left[4,i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}$$

Result (type 4, 614 leaves, 19 steps):

$$\frac{3\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{2\,x\,\sqrt{1-a^2\,x^2}}-\frac{(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{2\,x^2\,\sqrt{1-a^2\,x^2}}-$$
$$\frac{6\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+\frac{a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^3\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}-$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}+$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[4,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[4,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}$$

Problem 261: Result valid but suboptimal antiderivative.

$$\int \frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^3}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^3}{\sqrt{1-c^2\,x^2}},x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^3}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 267: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\,\sqrt{1-c^2\,x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$-\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{4\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^5\,\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{6\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^5\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{2\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}}-$$
$$\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{4\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^5\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{6\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}}$$

Result (type 4, 430 leaves, 13 steps):

$$-\frac{\sqrt{1-c^2\,x^2}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2\,x^2}\;\text{Cosh}\left[\frac{4\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^5\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\;\text{Cosh}\left[\frac{6\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2\,x^2}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^5\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\;\text{Sinh}\left[\frac{2\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2\,x^2}\;\text{Sinh}\left[\frac{4\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^5\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\;\text{Sinh}\left[\frac{6\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}$$

Problem 268: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,\sqrt{1-c^2\,x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 297 leaves, 12 steps):

$$-\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{a}{b}\right]\;\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^4\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{3\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{5\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{a}{b}\right]\;\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^4\,\sqrt{-1+c\,x}}-$$
$$\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{3\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{5\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}}$$

Result (type 4, 371 leaves, 13 steps):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,c^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{5\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,c^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{5\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 269: Result valid but suboptimal antiderivative.

$$\int \frac{x^2\,\sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{4\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{4\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{4\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{4\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 270: Result valid but suboptimal antiderivative.

$$\int \frac{x\,\sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$-\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}$$

Result (type 4, 245 leaves, 10 steps):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 271: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{2\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{2\,b\,c\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{2\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\sqrt{-1+c\,x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{2\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{2\,b\,c\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{2\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 272: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{x\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 116 leaves, 6 steps):

$$-\frac{\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b\sqrt{1-c\,x}}+$$
$$\frac{\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x\sqrt{1-c^2 x^2}\ (a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 176 leaves, 7 steps):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$

$$\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{1}{x\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 273: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^2\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 65 leaves, 3 steps):

$$-\frac{c\,\sqrt{-1+c\,x}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{b\,\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x^2\,\sqrt{1-c^2 x^2}\ (a+b\,\text{ArcCosh}[c\,x])},\,x\right]$$

Result (type 8, 115 leaves, 4 steps):

$$\frac{c\,\sqrt{1-c^2 x^2}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{1}{x^2\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 274: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 275: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^4\,(a+b\,\mathrm{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\mathrm{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^4\,(a+b\,\mathrm{ArcCosh}[c\,x])},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Unintegrable}\left[\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{x^4\,(a+b\,\mathrm{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 276: Result valid but suboptimal antiderivative.

$$\int \frac{x^3\,(1-c^2 x^2)^{3/2}}{a+b\,\mathrm{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$-\frac{3\sqrt{1-c\,x}\,\mathrm{Cosh}\left[\frac{a}{b}\right]\mathrm{CoshIntegral}\left[\frac{a+b\,\mathrm{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\,\mathrm{Cosh}\left[\frac{3a}{b}\right]\mathrm{CoshIntegral}\left[\frac{3\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\,\mathrm{Cosh}\left[\frac{5a}{b}\right]\mathrm{CoshIntegral}\left[\frac{5\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\mathrm{Cosh}\left[\frac{7a}{b}\right]\mathrm{CoshIntegral}\left[\frac{7\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+$$
$$\frac{3\sqrt{1-c\,x}\,\mathrm{Sinh}\left[\frac{a}{b}\right]\mathrm{SinhIntegral}\left[\frac{a+b\,\mathrm{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}-\frac{3\sqrt{1-c\,x}\,\mathrm{Sinh}\left[\frac{3a}{b}\right]\mathrm{SinhIntegral}\left[\frac{3\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}-$$
$$\frac{\sqrt{1-c\,x}\,\mathrm{Sinh}\left[\frac{5a}{b}\right]\mathrm{SinhIntegral}\left[\frac{5\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\mathrm{Sinh}\left[\frac{7a}{b}\right]\mathrm{SinhIntegral}\left[\frac{7\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}$$

Result (type 4, 497 leaves, 16 steps):

$$-\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{3a}{b}\right]\text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{5a}{b}\right]\text{CoshIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{7a}{b}\right]\text{CoshIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{3\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{3a}{b}\right]\text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{5a}{b}\right]\text{SinhIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{7a}{b}\right]\text{SinhIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 277: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\left(1-c^2x^2\right)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^3\sqrt{-1+c\,x}}-$$
$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6a}{b}\right]\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^3\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}-$$
$$\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^3\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{6a}{b}\right]\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}$$

Result (type 4, 430 leaves, 13 steps):

$$\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{2a}{b}\right]\ \text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{4a}{b}\right]\ \text{CoshIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{6a}{b}\right]\ \text{CoshIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{2a}{b}\right]\ \text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{4a}{b}\right]\ \text{SinhIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{6a}{b}\right]\ \text{SinhIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 278: Result valid but suboptimal antiderivative.

$$\int \frac{x\left(1-c^2x^2\right)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 297 leaves, 12 steps):

$$-\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^2\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{3a}{b}\right]\ \text{CoshIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{5a}{b}\right]\ \text{CoshIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^2\sqrt{-1+c\,x}}-\frac{3\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{3a}{b}\right]\ \text{SinhIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{5a}{b}\right]\ \text{SinhIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\sqrt{-1+c\,x}}$$

Result (type 4, 371 leaves, 13 steps):

$$-\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{3a}{b}\right]\ \text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{5a}{b}\right]\ \text{CoshIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{3\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{3a}{b}\right]\ \text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{5a}{b}\right]\ \text{SinhIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 279: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 239 leaves, 9 steps):

$$\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{4\,a}{b}\right]\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c\,\sqrt{-1+c\,x}}-\frac{3\,\sqrt{1-c\,x}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{2\,a}{b}\right]\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{4\,a}{b}\right]\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 304 leaves, 10 steps):

$$\frac{\sqrt{1-c^2 x^2}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\;\text{Cosh}\left[\frac{4\,a}{b}\right]\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}-\frac{3\,\sqrt{1-c^2 x^2}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\;\text{Sinh}\left[\frac{2\,a}{b}\right]\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\;\text{Sinh}\left[\frac{4\,a}{b}\right]\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}}$$

Problem 280: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 215 leaves, 15 steps):

$$-\frac{5\,\sqrt{-1+c\,x}\;\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\;\text{Cosh}\left[\frac{3\,a}{b}\right]\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,\sqrt{1-c\,x}}+\frac{5\,\sqrt{-1+c\,x}\;\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\;\text{Sinh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x\,\sqrt{1-c^2 x^2}\;(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 301 leaves, 16 steps):

$$\frac{5\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{4\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{3a}{b}\right]\text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{4\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{5\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{4\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{3a}{b}\right]\text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{4\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Unintegrable}\left[\frac{1}{x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 281: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2x^2\right)^{3/2}}{x^2\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 163 leaves, 9 steps):

$$\frac{c\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\sqrt{1-c\,x}}-\frac{3\,c\sqrt{-1+c\,x}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{2\,b\sqrt{1-c\,x}}-\frac{c\sqrt{-1+c\,x}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 240 leaves, 10 steps):

$$-\frac{c\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\,c\sqrt{1-c^2x^2}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{2\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Unintegrable}\left[\frac{1}{x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 282: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2x^2\right)^{3/2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2x^2\right)^{3/2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{(-1+c x)^{3/2} (1+c x)^{3/2}}{x^3 (a+b \text{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 283: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^4 \left(a+b \text{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{3/2}}{x^4 \left(a+b \text{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{(-1+c x)^{3/2} (1+c x)^{3/2}}{x^4 (a+b \text{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 284: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(1-c^2 x^2\right)^{5/2}}{a+b \text{ArcCosh}[c x]} \, dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$-\frac{3 \sqrt{1-c x}\ \text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{128 b c^4 \sqrt{-1+c x}}+\frac{\sqrt{1-c x}\ \text{Cosh}\left[\frac{3 a}{b}\right] \text{CoshIntegral}\left[\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{32 b c^4 \sqrt{-1+c x}}-$$
$$\frac{3 \sqrt{1-c x}\ \text{Cosh}\left[\frac{7 a}{b}\right] \text{CoshIntegral}\left[\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right]}{256 b c^4 \sqrt{-1+c x}}+\frac{\sqrt{1-c x}\ \text{Cosh}\left[\frac{9 a}{b}\right] \text{CoshIntegral}\left[\frac{9 (a+b \text{ArcCosh}[c x])}{b}\right]}{256 b c^4 \sqrt{-1+c x}}+$$
$$\frac{3 \sqrt{1-c x}\ \text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{128 b c^4 \sqrt{-1+c x}}-\frac{\sqrt{1-c x}\ \text{Sinh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{32 b c^4 \sqrt{-1+c x}}+$$
$$\frac{3 \sqrt{1-c x}\ \text{Sinh}\left[\frac{7 a}{b}\right] \text{SinhIntegral}\left[\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right]}{256 b c^4 \sqrt{-1+c x}}-\frac{\sqrt{1-c x}\ \text{Sinh}\left[\frac{9 a}{b}\right] \text{SinhIntegral}\left[\frac{9 (a+b \text{ArcCosh}[c x])}{b}\right]}{256 b c^4 \sqrt{-1+c x}}$$

Result (type 4, 497 leaves, 16 steps):

$$-\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{128\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{3a}{b}\right]\text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{7a}{b}\right]\text{CoshIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{9a}{b}\right]\text{CoshIntegral}\left[\frac{9a}{b}+9\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{128\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{3a}{b}\right]\text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{7a}{b}\right]\text{SinhIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{9a}{b}\right]\text{SinhIntegral}\left[\frac{9a}{b}+9\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 285: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\left(1-c^2x^2\right)^{5/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 439 leaves, 15 steps):

$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6a}{b}\right]\text{CoshIntegral}\left[\frac{6(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{8a}{b}\right]\text{CoshIntegral}\left[\frac{8(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{128\,b\,c^3\sqrt{-1+c\,x}}-\frac{5\sqrt{1-c\,x}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{128\,b\,c^3\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{6a}{b}\right]\text{SinhIntegral}\left[\frac{6(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{8a}{b}\right]\text{SinhIntegral}\left[\frac{8(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{128\,b\,c^3\sqrt{-1+c\,x}}$$

Result (type 4, 556 leaves, 16 steps):

$$\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{6a}{b}\right]\text{CoshIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{8a}{b}\right]\text{CoshIntegral}\left[\frac{8a}{b}+8\,\text{ArcCosh}[c\,x]\right]}{128\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{5\sqrt{1-c^2 x^2}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{128\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{6a}{b}\right]\text{SinhIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{8a}{b}\right]\text{SinhIntegral}\left[\frac{8a}{b}+8\,\text{ArcCosh}[c\,x]\right]}{128\,b\,c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 286: Result valid but suboptimal antiderivative.

$$\int\frac{x\left(1-c^2 x^2\right)^{5/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$-\frac{5\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+\frac{9\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3a}{b}\right]\text{CoshIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}-$$
$$\frac{5\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{5a}{b}\right]\text{CoshIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{7a}{b}\right]\text{CoshIntegral}\left[\frac{7(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+$$
$$\frac{5\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}-\frac{9\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{3a}{b}\right]\text{SinhIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+$$
$$\frac{5\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{5a}{b}\right]\text{SinhIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{7a}{b}\right]\text{SinhIntegral}\left[\frac{7(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}$$

Result (type 4, 497 leaves, 16 steps):

$$-\frac{5\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{9\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{3a}{b}\right]\ \text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{5\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{5a}{b}\right]\ \text{CoshIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{7a}{b}\right]\ \text{CoshIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{5\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{9\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{3a}{b}\right]\ \text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{5\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{5a}{b}\right]\ \text{SinhIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{7a}{b}\right]\ \text{SinhIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 287: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$\frac{15\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{2a}{b}\right]\ \text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\sqrt{-1+c\,x}}-\frac{3\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{4a}{b}\right]\ \text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{6a}{b}\right]\ \text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\sqrt{-1+c\,x}}-\frac{5\sqrt{1-c\,x}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c\sqrt{-1+c\,x}}-\frac{15\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{2a}{b}\right]\ \text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{4a}{b}\right]\ \text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{6a}{b}\right]\ \text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\sqrt{-1+c\,x}}$$

Result (type 4, 430 leaves, 13 steps):

$$\frac{15\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{2a}{b}\right]\ \text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{3\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{4a}{b}\right]\ \text{CoshIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{6a}{b}\right]\ \text{CoshIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{5\sqrt{1-c^2x^2}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{15\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{2a}{b}\right]\ \text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{3\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{4a}{b}\right]\ \text{SinhIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{6a}{b}\right]\ \text{SinhIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 288: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2x^2\right)^{5/2}}{x\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 309 leaves, 27 steps):

$$-\frac{11\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,\sqrt{1-c\,x}}+$$
$$\frac{7\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{3a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{5a}{b}\right]\ \text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}+$$
$$\frac{11\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,\sqrt{1-c\,x}}-\frac{7\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{3a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}+$$
$$\frac{\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{5a}{b}\right]\ \text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x\sqrt{1-c^2x^2}\ (a+b\,\text{ArcCosh}[c\,x])},\,x\right]$$

Result (type 8, 421 leaves, 28 steps):

$$\frac{11\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{7\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{3a}{b}\right]\text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{5a}{b}\right]\text{CoshIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{11\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{7\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{3a}{b}\right]\text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{5a}{b}\right]\text{SinhIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Unintegrable}\left[\frac{1}{x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 289: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2x^2\right)^{5/2}}{x^2\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 254 leaves, 18 steps):

$$\frac{c\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{b\sqrt{1-c\,x}}-\frac{c\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\sqrt{1-c\,x}}-\frac{15\,c\sqrt{-1+c\,x}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\sqrt{1-c\,x}}-\frac{c\sqrt{-1+c\,x}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{b\sqrt{1-c\,x}}+\frac{c\sqrt{-1+c\,x}\,\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 357 leaves, 19 steps):

$$-\frac{c\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{15\,c\sqrt{1-c^2x^2}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{c\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Unintegrable}\left[\frac{1}{x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 290: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{5/2}}{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{5/2}}{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \, \text{Unintegrable}\left[\frac{(-1+c x)^{5/2} (1+c x)^{5/2}}{x^3 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x} \, \sqrt{1+c x}}$$

Problem 291: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{5/2}}{x^4 \left(a+b \operatorname{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{5/2}}{x^4 \left(a+b \operatorname{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \, \text{Unintegrable}\left[\frac{(-1+c x)^{5/2} (1+c x)^{5/2}}{x^4 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x} \, \sqrt{1+c x}}$$

Problem 292: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{1-a^2 x^2} \, \operatorname{ArcCosh}[a x]} \, dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{\sqrt{-1+a x} \, \text{CoshIntegral}[2 \operatorname{ArcCosh}[a x]]}{2 a^5 \sqrt{1-a x}}+\frac{\sqrt{-1+a x} \, \text{CoshIntegral}[4 \operatorname{ArcCosh}[a x]]}{8 a^5 \sqrt{1-a x}}+\frac{3 \sqrt{-1+a x} \, \text{Log}[\operatorname{ArcCosh}[a x]]}{8 a^5 \sqrt{1-a x}}$$

Result (type 4, 137 leaves, 6 steps):

$$\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{CoshIntegral}[2\,\text{ArcCosh}[a\,x]]}{2\,a^5\,\sqrt{1-a^2\,x^2}}+\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{CoshIntegral}[4\,\text{ArcCosh}[a\,x]]}{8\,a^5\,\sqrt{1-a^2\,x^2}}+\frac{3\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{Log}[\text{ArcCosh}[a\,x]]}{8\,a^5\,\sqrt{1-a^2\,x^2}}$$

Problem 293: Result valid but suboptimal antiderivative.

$$\int\frac{x^3}{\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}\,dx$$

Optimal (type 4, 65 leaves, 5 steps):

$$\frac{3\,\sqrt{-1+a\,x}\;\text{CoshIntegral}[\text{ArcCosh}[a\,x]]}{4\,a^4\,\sqrt{1-a\,x}}+\frac{\sqrt{-1+a\,x}\;\text{CoshIntegral}[3\,\text{ArcCosh}[a\,x]]}{4\,a^4\,\sqrt{1-a\,x}}$$

Result (type 4, 91 leaves, 6 steps):

$$\frac{3\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{CoshIntegral}[\text{ArcCosh}[a\,x]]}{4\,a^4\,\sqrt{1-a^2\,x^2}}+\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{CoshIntegral}[3\,\text{ArcCosh}[a\,x]]}{4\,a^4\,\sqrt{1-a^2\,x^2}}$$

Problem 294: Result valid but suboptimal antiderivative.

$$\int\frac{x^2}{\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}\,dx$$

Optimal (type 4, 65 leaves, 4 steps):

$$\frac{\sqrt{-1+a\,x}\;\text{CoshIntegral}[2\,\text{ArcCosh}[a\,x]]}{2\,a^3\,\sqrt{1-a\,x}}+\frac{\sqrt{-1+a\,x}\;\text{Log}[\text{ArcCosh}[a\,x]]}{2\,a^3\,\sqrt{1-a\,x}}$$

Result (type 4, 91 leaves, 5 steps):

$$\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{CoshIntegral}[2\,\text{ArcCosh}[a\,x]]}{2\,a^3\,\sqrt{1-a^2\,x^2}}+\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{Log}[\text{ArcCosh}[a\,x]]}{2\,a^3\,\sqrt{1-a^2\,x^2}}$$

Problem 295: Result valid but suboptimal antiderivative.

$$\int\frac{x}{\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}\,dx$$

Optimal (type 4, 28 leaves, 2 steps):

$$\frac{\sqrt{-1+a\,x}\;\text{CoshIntegral}[\text{ArcCosh}[a\,x]]}{a^2\,\sqrt{1-a\,x}}$$

Result (type 4, 41 leaves, 3 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{CoshIntegral}[\text{ArcCosh}[a x]]}{a^2\sqrt{1-a^2 x^2}}$$

Problem 296: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]}\,dx$$

Optimal (type 3, 28 leaves, 1 step):

$$\frac{\sqrt{-1+a x}\ \text{Log}[\text{ArcCosh}[a x]]}{a\sqrt{1-a x}}$$

Result (type 3, 41 leaves, 2 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Log}[\text{ArcCosh}[a x]]}{a\sqrt{1-a^2 x^2}}$$

Problem 297: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]}\,dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]},\ x\right]$$

Result (type 8, 63 leaves, 1 step):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Unintegrable}\left[\frac{1}{x\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]},\ x\right]}{\sqrt{1-a^2 x^2}}$$

Problem 298: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]}\,dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 \sqrt{1-a^2 x^2}\, \text{ArcCosh}[a x]}, x\right]$$

Result (type 8, 63 leaves, 1 step):

$$\frac{\sqrt{-1+a x}\, \sqrt{1+a x}\, \text{Unintegrable}\left[\frac{1}{x^2 \sqrt{-1+a x}\, \sqrt{1+a x}\, \text{ArcCosh}[a x]}, x\right]}{\sqrt{1-a^2 x^2}}$$

Problem 299: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1-c^2 x^2}\, (a+b\, \text{ArcCosh}[c x])}\, dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$\frac{3 \sqrt{-1+c x}\, \text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a+b\, \text{ArcCosh}[c x]}{b}\right]}{4 b c^4 \sqrt{1-c x}} + \frac{\sqrt{-1+c x}\, \text{Cosh}\left[\frac{3 a}{b}\right] \text{CoshIntegral}\left[\frac{3 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{4 b c^4 \sqrt{1-c x}} - \frac{3 \sqrt{-1+c x}\, \text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b\, \text{ArcCosh}[c x]}{b}\right]}{4 b c^4 \sqrt{1-c x}} - \frac{\sqrt{-1+c x}\, \text{Sinh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{4 b c^4 \sqrt{1-c x}}$$

Result (type 4, 245 leaves, 10 steps):

$$\frac{3 \sqrt{-1+c x}\, \sqrt{1+c x}\, \text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{4 b c^4 \sqrt{1-c^2 x^2}} + \frac{\sqrt{-1+c x}\, \sqrt{1+c x}\, \text{Cosh}\left[\frac{3 a}{b}\right] \text{CoshIntegral}\left[\frac{3 a}{b}+3\, \text{ArcCosh}[c x]\right]}{4 b c^4 \sqrt{1-c^2 x^2}} - \frac{3 \sqrt{-1+c x}\, \sqrt{1+c x}\, \text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{4 b c^4 \sqrt{1-c^2 x^2}} - \frac{\sqrt{-1+c x}\, \sqrt{1+c x}\, \text{Sinh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 a}{b}+3\, \text{ArcCosh}[c x]\right]}{4 b c^4 \sqrt{1-c^2 x^2}}$$

Problem 300: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-c^2 x^2}\, (a+b\, \text{ArcCosh}[c x])}\, dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{-1+c x}\, \text{Cosh}\left[\frac{2 a}{b}\right] \text{CoshIntegral}\left[\frac{2 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{2 b c^3 \sqrt{1-c x}} + \frac{\sqrt{-1+c x}\, \text{Log}[a+b\, \text{ArcCosh}[c x]]}{2 b c^3 \sqrt{1-c x}} - \frac{\sqrt{-1+c x}\, \text{Sinh}\left[\frac{2 a}{b}\right] \text{SinhIntegral}\left[\frac{2 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{2 b c^3 \sqrt{1-c x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \text{Cosh}\left[\frac{2 a}{b}\right]\ \text{CoshIntegral}\left[\frac{2 a}{b}+2\ \text{ArcCosh}[c x]\right]}{2 b c^3 \sqrt{1-c^2 x^2}}+$$

$$\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \text{Log}[a+b\ \text{ArcCosh}[c x]]}{2 b c^3 \sqrt{1-c^2 x^2}}-\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \text{Sinh}\left[\frac{2 a}{b}\right]\ \text{SinhIntegral}\left[\frac{2 a}{b}+2\ \text{ArcCosh}[c x]\right]}{2 b c^3 \sqrt{1-c^2 x^2}}$$

Problem 301: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{1-c^2 x^2}\ (a+b\ \text{ArcCosh}[c x])}\, dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\frac{\sqrt{-1+c x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\ \text{ArcCosh}[c x]}{b}\right]}{b c^2 \sqrt{1-c x}}-\frac{\sqrt{-1+c x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\ \text{ArcCosh}[c x]}{b}\right]}{b c^2 \sqrt{1-c x}}$$

Result (type 4, 114 leaves, 5 steps):

$$\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{b c^2 \sqrt{1-c^2 x^2}}-\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{b c^2 \sqrt{1-c^2 x^2}}$$

Problem 302: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-c^2 x^2}\ (a+b\ \text{ArcCosh}[c x])}\, dx$$

Optimal (type 3, 35 leaves, 1 step):

$$\frac{\sqrt{-1+c x}\ \text{Log}[a+b\ \text{ArcCosh}[c x]]}{b c \sqrt{1-c x}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \text{Log}[a+b\ \text{ArcCosh}[c x]]}{b c \sqrt{1-c^2 x^2}}$$

Problem 303: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x \sqrt{1-c^2 x^2}\ (a+b\ \text{ArcCosh}[c x])}\, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 304: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 305: Result valid but suboptimal antiderivative.

$$\int\frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^2}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 306: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])} \, dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{\left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 66 leaves, 1 step):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x} \, \text{Unintegrable}\left[\frac{x}{(-1+c x)^{3/2} (1+c x)^{3/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 307: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])} \, dx$$

Optimal (type 8, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 65 leaves, 1 step):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x} \, \text{Unintegrable}\left[\frac{1}{(-1+c x)^{3/2} (1+c x)^{3/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 308: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x \left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x \left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 309: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\,\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\,\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x^2\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 310: Result valid but suboptimal antiderivative.

$$\int \frac{x^m\,\left(1-c^2\,x^2\right)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m\,\left(1-c^2\,x^2\right)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2\,x^2}\,\text{Unintegrable}\left[\frac{x^m\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]},\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 311: Result valid but suboptimal antiderivative.

$$\int \frac{x^m\,\sqrt{1-c^2\,x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m \sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{x^m \sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{a+b\,\text{ArcCosh}[c\,x]},x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 312: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{\sqrt{1-c^2 x^2}\ (a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{\sqrt{1-c^2 x^2}\ (a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ \text{Unintegrable}\left[\frac{x^m}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 313: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{\left(1-c^2 x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{\left(1-c^2 x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ \text{Unintegrable}\left[\frac{x^m}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 314: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{\left(1-c^2 x^2\right)^{5/2} (a+b \operatorname{ArcCosh}[c x])} \, dx$$

Optimal (type 8, 30 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{x^m}{\left(1-c^2 x^2\right)^{5/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step) :

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} \text{ Unintegrable}\left[\frac{x^m}{(-1+c x)^{5/2} (1+c x)^{5/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 320: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \sqrt{1-c^2 x^2}}{(a+b \operatorname{ArcCosh}[c x])^2} \, dx$$

Optimal (type 4, 350 leaves, 22 steps) :

$$-\frac{x^3 \sqrt{-1+c x} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{b c (a+b \operatorname{ArcCosh}[c x])}+\frac{\sqrt{1-c x} \text{ CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^4 \sqrt{-1+c x}}-\frac{3 \sqrt{1-c x} \text{ CoshIntegral}\left[\frac{3 (a+b \operatorname{ArcCosh}[c x])}{b}\right] \operatorname{Sinh}\left[\frac{3 a}{b}\right]}{16 b^2 c^4 \sqrt{-1+c x}}-$$
$$\frac{5 \sqrt{1-c x} \text{ CoshIntegral}\left[\frac{5 (a+b \operatorname{ArcCosh}[c x])}{b}\right] \operatorname{Sinh}\left[\frac{5 a}{b}\right]}{16 b^2 c^4 \sqrt{-1+c x}}-\frac{\sqrt{1-c x} \operatorname{Cosh}\left[\frac{a}{b}\right] \text{ SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{8 b^2 c^4 \sqrt{-1+c x}}+$$
$$\frac{3 \sqrt{1-c x} \operatorname{Cosh}\left[\frac{3 a}{b}\right] \text{ SinhIntegral}\left[\frac{3 (a+b \operatorname{ArcCosh}[c x])}{b}\right]}{16 b^2 c^4 \sqrt{-1+c x}}+\frac{5 \sqrt{1-c x} \operatorname{Cosh}\left[\frac{5 a}{b}\right] \text{ SinhIntegral}\left[\frac{5 (a+b \operatorname{ArcCosh}[c x])}{b}\right]}{16 b^2 c^4 \sqrt{-1+c x}}$$

Result (type 4, 429 leaves, 23 steps) :

$$\frac{x^3 (1-c x) \sqrt{1+c x} \sqrt{1-c^2 x^2}}{b c \sqrt{-1+c x} (a+b \text{ArcCosh}[c x])}+\frac{\sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^4 \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{3 \sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{3 a}{b}+3 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{3 a}{b}\right]}{16 b^2 c^4 \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{5 \sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{5 a}{b}+5 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{5 a}{b}\right]}{16 b^2 c^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{8 b^2 c^4 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{3 \sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 a}{b}+3 \text{ArcCosh}[c x]\right]}{16 b^2 c^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 \sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{5 a}{b}\right] \text{SinhIntegral}\left[\frac{5 a}{b}+5 \text{ArcCosh}[c x]\right]}{16 b^2 c^4 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \sqrt{1-c^2 x^2}}{(a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 4, 154 leaves, 16 steps):

$$-\frac{x^2 \sqrt{-1+c x} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{b c (a+b \text{ArcCosh}[c x])}-\frac{\sqrt{1-c x} \text{ CoshIntegral}\left[\frac{4 (a+b \text{ArcCosh}[c x])}{b}\right] \text{Sinh}\left[\frac{4 a}{b}\right]}{2 b^2 c^3 \sqrt{-1+c x}}+\frac{\sqrt{1-c x} \text{ Cosh}\left[\frac{4 a}{b}\right] \text{SinhIntegral}\left[\frac{4 (a+b \text{ArcCosh}[c x])}{b}\right]}{2 b^2 c^3 \sqrt{-1+c x}}$$

Result (type 4, 185 leaves, 17 steps):

$$\frac{x^2 (1-c x) \sqrt{1+c x} \sqrt{1-c^2 x^2}}{b c \sqrt{-1+c x} (a+b \text{ArcCosh}[c x])}-\frac{\sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{4 a}{b}\right]}{2 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{\sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{4 a}{b}\right] \text{SinhIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right]}{2 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int \frac{x \sqrt{1-c^2 x^2}}{(a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 4, 248 leaves, 14 steps):

$$-\frac{x\sqrt{-1+c x}\,\sqrt{1+c x}\,\sqrt{1-c^2 x^2}}{b c\,(a+b\,\text{ArcCosh}[c x])}+\frac{\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]\text{Sinh}\left[\frac{a}{b}\right]}{4 b^2 c^2\sqrt{-1+c x}}-\frac{3\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c x])}{b}\right]\text{Sinh}\left[\frac{3 a}{b}\right]}{4 b^2 c^2\sqrt{-1+c x}}-\frac{\sqrt{1-c x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]}{4 b^2 c^2\sqrt{-1+c x}}+\frac{3\sqrt{1-c x}\,\text{Cosh}\left[\frac{3 a}{b}\right]\text{SinhIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c x])}{b}\right]}{4 b^2 c^2\sqrt{-1+c x}}$$

Result (type 4, 418 leaves, 15 steps):

$$\frac{x(1-c x)\sqrt{1+c x}\,\sqrt{1-c^2 x^2}}{b c\sqrt{-1+c x}\,(a+b\,\text{ArcCosh}[c x])}-\frac{3\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]\text{Sinh}\left[\frac{a}{b}\right]}{4 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]\text{Sinh}\left[\frac{a}{b}\right]}{b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{3\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{3 a}{b}+3\,\text{ArcCosh}[c x]\right]\text{Sinh}\left[\frac{3 a}{b}\right]}{4 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{3\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{4 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{3\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{3 a}{b}\right]\text{SinhIntegral}\left[\frac{3 a}{b}+3\,\text{ArcCosh}[c x]\right]}{4 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]}{b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$-\frac{\sqrt{-1+c x}\,\sqrt{1+c x}\,\sqrt{1-c^2 x^2}}{b c\,(a+b\,\text{ArcCosh}[c x])}-\frac{\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]\text{Sinh}\left[\frac{2 a}{b}\right]}{b^2 c\sqrt{-1+c x}}+\frac{\sqrt{1-c x}\,\text{Cosh}\left[\frac{2 a}{b}\right]\text{SinhIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]}{b^2 c\sqrt{-1+c x}}$$

Result (type 4, 177 leaves, 8 steps):

$$\frac{(1-c x)\sqrt{1+c x}\,\sqrt{1-c^2 x^2}}{b c\sqrt{-1+c x}\,(a+b\,\text{ArcCosh}[c x])}-\frac{\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{2 a}{b}+2\,\text{ArcCosh}[c x]\right]\text{Sinh}\left[\frac{2 a}{b}\right]}{b^2 c\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{2 a}{b}\right]\text{SinhIntegral}\left[\frac{2 a}{b}+2\,\text{ArcCosh}[c x]\right]}{b^2 c\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 325: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{x^2\,(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 8, 97 leaves, 1 step):

$$-\frac{\sqrt{-1+c x}\,\sqrt{1+c x}\,\sqrt{1-c^2 x^2}}{b c x^2\,(a+b\,\text{ArcCosh}[c x])}+\frac{2\sqrt{1-c x}\ \text{Unintegrable}\left[\frac{1}{x^3\,(a+b\,\text{ArcCosh}[c x])},\ x\right]}{b c\sqrt{-1+c x}}$$

Result (type 8, 115 leaves, 2 steps):

$$\frac{(1-c x)\,\sqrt{1+c x}\,\sqrt{1-c^2 x^2}}{b c x^2\sqrt{-1+c x}\,(a+b\,\text{ArcCosh}[c x])}+\frac{2\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{1}{x^3\,(a+b\,\text{ArcCosh}[c x])},\ x\right]}{b c\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 326: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{x^3\,(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^3\,(a+b\,\text{ArcCosh}[c x])^2},\ x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{\sqrt{-1+c x}\,\sqrt{1+c x}}{x^3\,(a+b\,\text{ArcCosh}[c x])^2},\ x\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 327: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{x^4\,(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^4\,(a+b\,\text{ArcCosh}[c x])^2},\ x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{\sqrt{-1+c x}\,\sqrt{1+c x}}{x^4\,(a+b\,\text{ArcCosh}[c x])^2},\ x\right]}{\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 328: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(1-c^2 x^2\right)^{3/2}}{(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 4, 354 leaves, 21 steps):

$$-\frac{x^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2 x^2\right)^{3/2}}{b\,c\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{4\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b^2\,c^3\,\sqrt{-1+c\,x}}-\frac{3\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 439 leaves, 20 steps):

$$\frac{x^2\,(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2 x^2}}{b\,c\,\sqrt{-1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{4\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{4\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3\,\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(1-c^2 x^2\right)^{3/2}}{(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 4, 348 leaves, 24 steps):

$$-\frac{x\sqrt{-1+c x}\,\sqrt{1+c x}\,\left(1-c^2 x^2\right)^{3/2}}{b c\,(a+b\,\text{ArcCosh}[c x])}+\frac{\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]\text{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^2\sqrt{-1+c x}}-\frac{9\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c x])}{b}\right]\text{Sinh}\left[\frac{3 a}{b}\right]}{16 b^2 c^2\sqrt{-1+c x}}+\frac{5\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c x])}{b}\right]\text{Sinh}\left[\frac{5 a}{b}\right]}{16 b^2 c^2\sqrt{-1+c x}}-\frac{\sqrt{1-c x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]}{8 b^2 c^2\sqrt{-1+c x}}+\frac{9\sqrt{1-c x}\,\text{Cosh}\left[\frac{3 a}{b}\right]\text{SinhIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c x])}{b}\right]}{16 b^2 c^2\sqrt{-1+c x}}-\frac{5\sqrt{1-c x}\,\text{Cosh}\left[\frac{5 a}{b}\right]\text{SinhIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c x])}{b}\right]}{16 b^2 c^2\sqrt{-1+c x}}$$

Result (type 4, 429 leaves, 23 steps):

$$\frac{x\,(1-c x)^2\,(1+c x)^{3/2}\sqrt{1-c^2 x^2}}{b c\sqrt{-1+c x}\,(a+b\,\text{ArcCosh}[c x])}+\frac{\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]\text{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{9\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{3 a}{b}+3\,\text{ArcCosh}[c x]\right]\text{Sinh}\left[\frac{3 a}{b}\right]}{16 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{5\sqrt{1-c^2 x^2}\,\text{CoshIntegral}\left[\frac{5 a}{b}+5\,\text{ArcCosh}[c x]\right]\text{Sinh}\left[\frac{5 a}{b}\right]}{16 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{8 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{9\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{3 a}{b}\right]\text{SinhIntegral}\left[\frac{3 a}{b}+3\,\text{ArcCosh}[c x]\right]}{16 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{5\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{5 a}{b}\right]\text{SinhIntegral}\left[\frac{5 a}{b}+5\,\text{ArcCosh}[c x]\right]}{16 b^2 c^2\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 330: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2 x^2\right)^{3/2}}{(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 4, 246 leaves, 11 steps):

$$-\frac{\sqrt{-1+c x}\,\sqrt{1+c x}\,\left(1-c^2 x^2\right)^{3/2}}{b c\,(a+b\,\text{ArcCosh}[c x])}-\frac{\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]\text{Sinh}\left[\frac{2 a}{b}\right]}{b^2 c\sqrt{-1+c x}}+\frac{\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c x])}{b}\right]\text{Sinh}\left[\frac{4 a}{b}\right]}{2 b^2 c\sqrt{-1+c x}}+\frac{\sqrt{1-c x}\,\text{Cosh}\left[\frac{2 a}{b}\right]\text{SinhIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]}{b^2 c\sqrt{-1+c x}}-\frac{\sqrt{1-c x}\,\text{Cosh}\left[\frac{4 a}{b}\right]\text{SinhIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c x])}{b}\right]}{2 b^2 c\sqrt{-1+c x}}$$

Result (type 4, 305 leaves, 11 steps):

$$\frac{(1-c x)^2 (1+c x)^{3/2} \sqrt{1-c^2 x^2}}{b c \sqrt{-1+c x} (a+b \text{ArcCosh}[c x])}-\frac{\sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{2 a}{b}+2 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{2 a}{b}\right]}{b^2 c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{\sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{4 a}{b}\right]}{2 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{\sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{2 a}{b}\right] \text{SinhIntegral}\left[\frac{2 a}{b}+2 \text{ArcCosh}[c x]\right]}{b^2 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{4 a}{b}\right] \text{SinhIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right]}{2 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 332: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^2 (a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 156 leaves, 3 steps):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x} \left(1-c^2 x^2\right)^{3/2}}{b c x^2 (a+b \text{ArcCosh}[c x])}-\frac{2 \sqrt{1-c x} \text{ Unintegrable}\left[\frac{-1+c^2 x^2}{x^3 (a+b \text{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+c x}}-\frac{2 c \sqrt{1-c x} \text{ Unintegrable}\left[\frac{-1+c^2 x^2}{x (a+b \text{ArcCosh}[c x])}, x\right]}{b \sqrt{-1+c x}}$$

Result (type 8, 189 leaves, 2 steps):

$$\frac{(1-c x)^2 (1+c x)^{3/2} \sqrt{1-c^2 x^2}}{b c x^2 \sqrt{-1+c x} (a+b \text{ArcCosh}[c x])}-\frac{2 \sqrt{1-c^2 x^2} \text{ Unintegrable}\left[\frac{-1+c^2 x^2}{x^3 (a+b \text{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 c \sqrt{1-c^2 x^2} \text{ Unintegrable}\left[\frac{-1+c^2 x^2}{x (a+b \text{ArcCosh}[c x])}, x\right]}{b \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 333: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^3 (a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{3/2}}{x^3 (a+b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2} \text{ Unintegrable}\left[\frac{(-1+c x)^{3/2} (1+c x)^{3/2}}{x^3 (a+b \text{ArcCosh}[c x])^2}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 334: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^4 \left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 106 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2 x^2\right)^{3/2}}{b\,c\,x^4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{4\,\sqrt{1-c\,x}\,\text{Unintegrable}\left[\frac{-1+c^2 x^2}{x^5\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,c\,\sqrt{-1+c\,x}}$$

Result (type 8, 126 leaves, 2 steps):

$$\frac{(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2 x^2}}{b\,c\,x^4\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{4\,\sqrt{1-c^2 x^2}\,\text{Unintegrable}\left[\frac{-1+c^2 x^2}{x^5\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 335: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(1-c^2 x^2\right)^{5/2}}{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 454 leaves, 30 steps):

$$-\frac{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2 x^2\right)^{5/2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{8\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$

$$\frac{3\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{8\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$

$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b^2\,c^3\,\sqrt{-1+c\,x}}-$$

$$\frac{3\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{8\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 565 leaves, 29 steps):

$$\frac{x^2 (1-c x)^3 (1+c x)^{5/2} \sqrt{1-c^2 x^2}}{b c \sqrt{-1+c x} (a+b \text{ArcCosh}[c x])}-\frac{\sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{2 a}{b}+2 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{2 a}{b}\right]}{16 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{\sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{4 a}{b}\right]}{8 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{3 \sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{6 a}{b}+6 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{6 a}{b}\right]}{16 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{8 a}{b}+8 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{8 a}{b}\right]}{16 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{\sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{2 a}{b}\right] \text{SinhIntegral}\left[\frac{2 a}{b}+2 \text{ArcCosh}[c x]\right]}{16 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{\sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{4 a}{b}\right] \text{SinhIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right]}{8 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{3 \sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{6 a}{b}\right] \text{SinhIntegral}\left[\frac{6 a}{b}+6 \text{ArcCosh}[c x]\right]}{16 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{\sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{8 a}{b}\right] \text{SinhIntegral}\left[\frac{8 a}{b}+8 \text{ArcCosh}[c x]\right]}{16 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 336: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(1-c^2 x^2\right)^{5/2}}{(a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 4, 448 leaves, 30 steps):

$$-\frac{x \sqrt{-1+c x} \sqrt{1+c x} \left(1-c^2 x^2\right)^{5/2}}{b c (a+b \text{ArcCosh}[c x])}+\frac{5 \sqrt{1-c x} \text{ CoshIntegral}\left[\frac{a+b \text{ArcCosh}[c x]}{b}\right] \text{Sinh}\left[\frac{a}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}-\frac{27 \sqrt{1-c x} \text{ CoshIntegral}\left[\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right] \text{Sinh}\left[\frac{3 a}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}+$$

$$\frac{25 \sqrt{1-c x} \text{ CoshIntegral}\left[\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right] \text{Sinh}\left[\frac{5 a}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}-\frac{7 \sqrt{1-c x} \text{ CoshIntegral}\left[\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right] \text{Sinh}\left[\frac{7 a}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}-$$

$$\frac{5 \sqrt{1-c x} \text{ Cosh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}+\frac{27 \sqrt{1-c x} \text{ Cosh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}-$$

$$\frac{25 \sqrt{1-c x} \text{ Cosh}\left[\frac{5 a}{b}\right] \text{SinhIntegral}\left[\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}+\frac{7 \sqrt{1-c x} \text{ Cosh}\left[\frac{7 a}{b}\right] \text{SinhIntegral}\left[\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right]}{64 b^2 c^2 \sqrt{-1+c x}}$$

Result (type 4, 555 leaves, 29 steps):

$$\frac{x\,(1-c\,x)^3\,(1+c\,x)^{5/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{5\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{27\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{25\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{5\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{7\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{7\,a}{b}+7\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{7\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{5\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{27\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{25\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{7\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{7\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{7\,a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 337: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2\,x^2\right)^{5/2}}{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 351 leaves, 14 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)^{5/2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{15\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}+\frac{3\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{4\,b^2\,c\,\sqrt{-1+c\,x}}-$$
$$\frac{3\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}+\frac{15\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}-$$
$$\frac{3\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b^2\,c\,\sqrt{-1+c\,x}}+\frac{3\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 436 leaves, 14 steps):

$$\frac{(1-c x)^3 (1+c x)^{5/2} \sqrt{1-c^2 x^2}}{b c \sqrt{-1+c x} (a+b \text{ArcCosh}[c x])}-\frac{15 \sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{2 a}{b}+2 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{2 a}{b}\right]}{16 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{3 \sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{4 a}{b}\right]}{4 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{3 \sqrt{1-c^2 x^2} \text{ CoshIntegral}\left[\frac{6 a}{b}+6 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{6 a}{b}\right]}{16 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{15 \sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{2 a}{b}\right] \text{SinhIntegral}\left[\frac{2 a}{b}+2 \text{ArcCosh}[c x]\right]}{16 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{3 \sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{4 a}{b}\right] \text{SinhIntegral}\left[\frac{4 a}{b}+4 \text{ArcCosh}[c x]\right]}{4 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}+\frac{3 \sqrt{1-c^2 x^2} \text{ Cosh}\left[\frac{6 a}{b}\right] \text{SinhIntegral}\left[\frac{6 a}{b}+6 \text{ArcCosh}[c x]\right]}{16 b^2 c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 339: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{5/2}}{x^2 (a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 160 leaves, 3 steps):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x} \left(1-c^2 x^2\right)^{5/2}}{b c x^2 (a+b \text{ArcCosh}[c x])}+\frac{2 \sqrt{1-c x} \text{ Unintegrable}\left[\frac{\left(-1+c^2 x^2\right)^2}{x^3 (a+b \text{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+c x}}+\frac{4 c \sqrt{1-c x} \text{ Unintegrable}\left[\frac{\left(-1+c^2 x^2\right)^2}{x (a+b \text{ArcCosh}[c x])}, x\right]}{b \sqrt{-1+c x}}$$

Result (type 8, 193 leaves, 2 steps):

$$\frac{(1-c x)^3 (1+c x)^{5/2} \sqrt{1-c^2 x^2}}{b c x^2 \sqrt{-1+c x} (a+b \text{ArcCosh}[c x])}+\frac{2 \sqrt{1-c^2 x^2} \text{ Unintegrable}\left[\frac{\left(-1+c^2 x^2\right)^2}{x^3 (a+b \text{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+c x} \sqrt{1+c x}}+\frac{4 c \sqrt{1-c^2 x^2} \text{ Unintegrable}\left[\frac{\left(-1+c^2 x^2\right)^2}{x (a+b \text{ArcCosh}[c x])}, x\right]}{b \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 340: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{5/2}}{x^3 (a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{5/2}}{x^3 (a+b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2x^2}\ \text{Unintegrable}\left[\frac{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 341: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2x^2\right)^{5/2}}{x^4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2x^2}\ \text{Unintegrable}\left[\frac{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}}{x^4\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 342: Result valid but suboptimal antiderivative.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}\ \left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 337 leaves, 13 steps):

$$-\frac{x^5\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\ \left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{5\sqrt{-1+c\,x}\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2\,c^6\,\sqrt{1-c\,x}}-\frac{15\sqrt{-1+c\,x}\ \text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2\,c^6\,\sqrt{1-c\,x}}-$$
$$\frac{5\sqrt{-1+c\,x}\ \text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2\,c^6\,\sqrt{1-c\,x}}+\frac{5\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b^2\,c^6\,\sqrt{1-c\,x}}+$$
$$\frac{15\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^6\,\sqrt{1-c\,x}}+\frac{5\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{5\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^6\,\sqrt{1-c\,x}}$$

Result (type 4, 424 leaves, 14 steps):

$$-\frac{x^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2\,c^6\,\sqrt{1-c^2\,x^2}}-$$
$$\frac{15\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2\,c^6\,\sqrt{1-c^2\,x^2}}-$$
$$\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2\,c^6\,\sqrt{1-c^2\,x^2}}+\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b^2\,c^6\,\sqrt{1-c^2\,x^2}}+$$
$$\frac{15\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^6\,\sqrt{1-c^2\,x^2}}+\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^6\,\sqrt{1-c^2\,x^2}}$$

Problem 343: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 236 leaves, 10 steps):

$$-\frac{x^4\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c^5\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c^5\,\sqrt{1-c\,x}}+$$
$$\frac{\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{b^2\,c^5\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b^2\,c^5\,\sqrt{1-c\,x}}$$

Result (type 4, 301 leaves, 11 steps):

$$-\frac{x^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c^5\,\sqrt{1-c^2\,x^2}}-$$
$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c^5\,\sqrt{1-c^2\,x^2}}+$$
$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b^2\,c^5\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{2\,b^2\,c^5\,\sqrt{1-c^2\,x^2}}$$

Problem 344: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 237 leaves, 10 steps):

$$-\frac{x^3\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{3\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\text{Sinh}\left[\frac{a}{b}\right]}{4\,b^2\,c^4\,\sqrt{1-c\,x}}-\frac{3\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\text{Sinh}\left[\frac{3\,a}{b}\right]}{4\,b^2\,c^4\,\sqrt{1-c\,x}}+\frac{3\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b^2\,c^4\,\sqrt{1-c\,x}}+\frac{3\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b^2\,c^4\,\sqrt{1-c\,x}}$$

Result (type 4, 298 leaves, 11 steps):

$$-\frac{x^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{a}{b}\right]}{4\,b^2\,c^4\,\sqrt{1-c^2 x^2}}-\frac{3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{3\,a}{b}\right]}{4\,b^2\,c^4\,\sqrt{1-c^2 x^2}}+\frac{3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{4\,b^2\,c^4\,\sqrt{1-c^2 x^2}}+\frac{3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{4\,b^2\,c^4\,\sqrt{1-c^2 x^2}}$$

Problem 345: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 136 leaves, 7 steps):

$$-\frac{x^2\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c^3\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{b^2\,c^3\,\sqrt{1-c\,x}}$$

Result (type 4, 175 leaves, 8 steps):

$$-\frac{x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c^3\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b^2\,c^3\,\sqrt{1-c^2\,x^2}}$$

Problem 346: Result valid but suboptimal antiderivative.

$$\int\frac{x}{\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{x\,\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{b^2\,c^2\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b^2\,c^2\,\sqrt{1-c\,x}}$$

Result (type 4, 169 leaves, 6 steps):

$$-\frac{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{b^2\,c^2\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b^2\,c^2\,\sqrt{1-c^2\,x^2}}$$

Problem 347: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 3, 37 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}$$

Result (type 3, 50 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}$$

Problem 348: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x \sqrt{1-c^2 x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^2} \, dx$$

Optimal (type 8, 84 leaves, 1 step):

$$-\frac{\sqrt{-1+c x}}{b c x \sqrt{1-c x} \left(a+b \operatorname{ArcCosh}[c x]\right)}-\frac{\sqrt{-1+c x} \operatorname{Unintegrable}\left[\frac{1}{x^2 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{1-c x}}$$

Result (type 8, 110 leaves, 2 steps):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x}}{b c x \sqrt{1-c^2 x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)}-\frac{\sqrt{-1+c x} \sqrt{1+c x} \operatorname{Unintegrable}\left[\frac{1}{x^2 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{1-c^2 x^2}}$$

Problem 349: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \sqrt{1-c^2 x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^2} \, dx$$

Optimal (type 8, 84 leaves, 1 step):

$$-\frac{\sqrt{-1+c x}}{b c x^2 \sqrt{1-c x} \left(a+b \operatorname{ArcCosh}[c x]\right)}-\frac{2 \sqrt{-1+c x} \operatorname{Unintegrable}\left[\frac{1}{x^3 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{1-c x}}$$

Result (type 8, 110 leaves, 2 steps):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x}}{b c x^2 \sqrt{1-c^2 x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)}-\frac{2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{Unintegrable}\left[\frac{1}{x^3 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{1-c^2 x^2}}$$

Problem 350: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(1-c^2 x^2\right)^{3/2} \left(a+b \operatorname{ArcCosh}[c x]\right)^2} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{x^3}{\left(1-c^2 x^2\right)^{3/2} \left(a+b \operatorname{ArcCosh}[c x]\right)^2}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^3}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 351: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 106 leaves, 2 steps):

$$-\frac{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{2\,\sqrt{-1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2\,x^2\right)^2\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,c\,\sqrt{1-c\,x}}$$

Result (type 8, 127 leaves, 2 steps):

$$-\frac{x^2\,\sqrt{-1+c\,x}}{b\,c\,(1-c\,x)\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2\,x^2\right)^2\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,c\,\sqrt{1-c^2\,x^2}}$$

Problem 352: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 66 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 353: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 101 leaves, 2 steps):

$$-\frac{\sqrt{-1+c x}\,\sqrt{1+c x}}{b c\left(1-c^2 x^2\right)^{3/2}(a+b\,\text{ArcCosh}[c x])}+\frac{2 c \sqrt{-1+c x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2 x^2\right)^2 (a+b\,\text{ArcCosh}[c x])},x\right]}{b \sqrt{1-c x}}$$

Result (type 8, 122 leaves, 2 steps):

$$-\frac{\sqrt{-1+c x}}{b c (1-c x) \sqrt{1+c x}\,\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c x])}+\frac{2 c \sqrt{-1+c x}\,\sqrt{1+c x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2 x^2\right)^2 (a+b\,\text{ArcCosh}[c x])},x\right]}{b \sqrt{1-c^2 x^2}}$$

Problem 354: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\left(1-c^2 x^2\right)^{3/2}(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\left(1-c^2 x^2\right)^{3/2}(a+b\,\text{ArcCosh}[c x])^2},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{Unintegrable}\left[\frac{1}{x (-1+c x)^{3/2} (1+c x)^{3/2} (a+b\,\text{ArcCosh}[c x])^2},x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 355: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\left(1-c^2 x^2\right)^{3/2}(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\left(1-c^2 x^2\right)^{3/2}(a+b\,\text{ArcCosh}[c x])^2},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{Unintegrable}\left[\frac{1}{x^2 (-1+c x)^{3/2} (1+c x)^{3/2} (a+b\,\text{ArcCosh}[c x])^2},x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 356: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(1-c^2 x^2\right)^{5/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\, dx$$

Optimal (type 8, 108 leaves, 2 steps):

$$-\frac{x^4 \sqrt{-1+c\,x}\, \sqrt{1+c\,x}}{b\,c\,\left(1-c^2 x^2\right)^{5/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{4 \sqrt{-1+c\,x}\ \text{Unintegrable}\left[\frac{x^3}{\left(-1+c^2 x^2\right)^3 \left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]}{b\,c\,\sqrt{1-c\,x}}$$

Result (type 8, 129 leaves, 2 steps):

$$-\frac{x^4 \sqrt{-1+c\,x}}{b\,c\,\left(1-c\,x\right)^2 \left(1+c\,x\right)^{3/2} \sqrt{1-c^2 x^2}\, \left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{4 \sqrt{-1+c\,x}\, \sqrt{1+c\,x}\ \text{Unintegrable}\left[\frac{x^3}{\left(-1+c^2 x^2\right)^3 \left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]}{b\,c\,\sqrt{1-c^2 x^2}}$$

Problem 357: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(1-c^2 x^2\right)^{5/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^3}{\left(1-c^2 x^2\right)^{5/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\, \sqrt{1+c\,x}\ \text{Unintegrable}\left[\frac{x^3}{\left(-1+c\,x\right)^{5/2} \left(1+c\,x\right)^{5/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 358: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\left(1-c^2 x^2\right)^{5/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{\left(1-c^2 x^2\right)^{5/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^2}{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 359: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 65 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 360: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 101 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{4\,c\,\sqrt{-1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2\,x^2\right)^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,\sqrt{1-c\,x}}$$

Result (type 8, 122 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}}{b\,c\,(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{4\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2\,x^2\right)^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,\sqrt{1-c^2\,x^2}}$$

Problem 361: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\,\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x\,(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2x^2}}$$

Problem 362: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x^2\,(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2x^2}}$$

Problem 363: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{\sqrt{1-c^2x^2}\,\text{Unintegrable}\left[\frac{(f\,x)^m\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}}{(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 364: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m \sqrt{1-c^2 x^2}}{(a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m \sqrt{1-c^2 x^2}}{(a+b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \, \text{Unintegrable}\left[\frac{(f x)^m \sqrt{-1+c x} \sqrt{1+c x}}{(a+b \text{ArcCosh}[c x])^2}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 365: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m}{\sqrt{1-c^2 x^2} \, (a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 91 leaves, 1 step):

$$-\frac{(f x)^m \sqrt{-1+c x}}{b c \sqrt{1-c x} \, (a+b \text{ArcCosh}[c x])}+\frac{f m \sqrt{-1+c x} \, \text{Unintegrable}\left[\frac{(f x)^{-1+m}}{a+b \text{ArcCosh}[c x]}, x\right]}{b c \sqrt{1-c x}}$$

Result (type 8, 117 leaves, 2 steps):

$$-\frac{(f x)^m \sqrt{-1+c x} \sqrt{1+c x}}{b c \sqrt{1-c^2 x^2} \, (a+b \text{ArcCosh}[c x])}+\frac{f m \sqrt{-1+c x} \sqrt{1+c x} \, \text{Unintegrable}\left[\frac{(f x)^{-1+m}}{a+b \text{ArcCosh}[c x]}, x\right]}{b c \sqrt{1-c^2 x^2}}$$

Problem 366: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m}{(1-c^2 x^2)^{3/2} \, (a+b \text{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m}{(1-c^2 x^2)^{3/2} \, (a+b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(f\,x)^m}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 367: Result valid but suboptimal antiderivative.

$$\int \frac{(f\,x)^m}{\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m}{\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(f\,x)^m}{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 368: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^3}\,dx$$

Optimal (type 3, 32 leaves, 1 step):

$$-\frac{\sqrt{-1+a\,x}}{2\,a\,\sqrt{1-a\,x}\,\text{ArcCosh}[a\,x]^2}$$

Result (type 3, 45 leaves, 2 steps):

$$-\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{2\,a\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}$$

Problem 380: Result optimal but 1 more steps used.

$$\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}\,dx$$

Optimal (type 4, 205 leaves, 10 steps):

$$\frac{1}{2} x \sqrt{c - a^2 c x^2} \sqrt{\text{ArcCosh}[a x]} - \frac{\sqrt{c - a^2 c x^2} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{-1 + a x} \sqrt{1 + a x}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1 + a x} \sqrt{1 + a x}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1 + a x} \sqrt{1 + a x}}$$

Result (type 4, 205 leaves, 11 steps):

$$\frac{1}{2} x \sqrt{c - a^2 c x^2} \sqrt{\text{ArcCosh}[a x]} - \frac{\sqrt{c - a^2 c x^2} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{-1 + a x} \sqrt{1 + a x}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1 + a x} \sqrt{1 + a x}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1 + a x} \sqrt{1 + a x}}$$

Problem 381: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}[a x]}}{\sqrt{c - a^2 c x^2}} \, dx$$

Optimal (type 3, 48 leaves, 1 step):

$$\frac{2 \sqrt{-1 + a x} \sqrt{1 + a x} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{c - a^2 c x^2}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{2 \sqrt{-1 + a x} \sqrt{1 + a x} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{c - a^2 c x^2}}$$

Problem 382: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}[a x]}}{\left(c - a^2 c x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 94 leaves, 1 step):

$$\frac{x \sqrt{\text{ArcCosh}[a x]}}{c \sqrt{c - a^2 c x^2}} + \frac{a \sqrt{-1 + a x} \sqrt{1 + a x} \text{Unintegrable}\left[\frac{x}{\left(1 - a^2 x^2\right) \sqrt{\text{ArcCosh}[a x]}}, x\right]}{2 c \sqrt{c - a^2 c x^2}}$$

Result (type 8, 94 leaves, 2 steps):

$$\frac{x\sqrt{\text{ArcCosh}[a\,x]}}{c\sqrt{c-a^2\,c\,x^2}}+\frac{a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Unintegrable}\left[\frac{x}{(1-a^2\,x^2)\sqrt{\text{ArcCosh}[a\,x]}},x\right]}{2\,c\sqrt{c-a^2\,c\,x^2}}$$

Problem 385: Result optimal but 1 more steps used.

$$\int\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}\,dx$$

Optimal (type 4, 302 leaves, 11 steps):

$$\frac{3\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{16\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{3\,a\,x^2\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{8\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}}{5\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Result (type 4, 302 leaves, 12 steps):

$$\frac{3\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{16\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{3\,a\,x^2\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{8\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}}{5\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 386: Result optimal but 1 more steps used.

$$\int\frac{\text{ArcCosh}[a\,x]^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Optimal (type 3, 48 leaves, 1 step):

$$\frac{2\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^{5/2}}{5\,a\sqrt{c-a^2\,c\,x^2}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{2\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^{5/2}}{5\,a\sqrt{c-a^2\,c\,x^2}}$$

Problem 387: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a\,x]^{3/2}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 8, 94 leaves, 1 step):

$$\frac{x\,\text{ArcCosh}[a\,x]^{3/2}}{c\,\sqrt{c-a^2\,c\,x^2}}+\frac{3\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Unintegrable}\left[\frac{x\,\sqrt{\text{ArcCosh}[a\,x]}}{1-a^2\,x^2},\,x\right]}{2\,c\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 8, 94 leaves, 2 steps):

$$\frac{x\,\text{ArcCosh}[a\,x]^{3/2}}{c\,\sqrt{c-a^2\,c\,x^2}}+\frac{3\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Unintegrable}\left[\frac{x\,\sqrt{\text{ArcCosh}[a\,x]}}{1-a^2\,x^2},\,x\right]}{2\,c\,\sqrt{c-a^2\,c\,x^2}}$$

Problem 389: Result optimal but 1 more steps used.

$$\int \sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}\,dx$$

Optimal (type 4, 330 leaves, 13 steps):

$$\frac{15}{32}\,x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}+\frac{5\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{16\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{5\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{8\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{7/2}}{7\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Result (type 4, 330 leaves, 14 steps):

$$\frac{15}{32}\,x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}+\frac{5\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{16\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{5\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{8\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{7/2}}{7\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 390: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a\,x]^{5/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Optimal (type 3, 48 leaves, 1 step):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^{7/2}}{7 a \sqrt{c-a^2 c x^2}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^{7/2}}{7 a \sqrt{c-a^2 c x^2}}$$

Problem 391: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a x]^{5/2}}{\left(c-a^2 c x^2\right)^{3/2}}\, dx$$

Optimal (type 8, 94 leaves, 1 step):

$$\frac{x\ \text{ArcCosh}[a x]^{5/2}}{c\sqrt{c-a^2 c x^2}}+\frac{5 a \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Unintegrable}\left[\frac{x\ \text{ArcCosh}[a x]^{3/2}}{1-a^2 x^2}, x\right]}{2 c \sqrt{c-a^2 c x^2}}$$

Result (type 8, 94 leaves, 2 steps):

$$\frac{x\ \text{ArcCosh}[a x]^{5/2}}{c\sqrt{c-a^2 c x^2}}+\frac{5 a \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Unintegrable}\left[\frac{x\ \text{ArcCosh}[a x]^{3/2}}{1-a^2 x^2}, x\right]}{2 c \sqrt{c-a^2 c x^2}}$$

Problem 393: Result optimal but 1 more steps used.

$$\int \sqrt{a^2-x^2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\, dx$$

Optimal (type 4, 211 leaves, 10 steps):

$$\frac{1}{2} x \sqrt{a^2-x^2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}-\frac{a\sqrt{a^2-x^2}\ \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3\sqrt{-1+\frac{x}{a}}\ \sqrt{1+\frac{x}{a}}}+\frac{a\sqrt{\frac{\pi}{2}}\ \sqrt{a^2-x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{16\sqrt{-1+\frac{x}{a}}\ \sqrt{1+\frac{x}{a}}}-\frac{a\sqrt{\frac{\pi}{2}}\ \sqrt{a^2-x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{16\sqrt{-1+\frac{x}{a}}\ \sqrt{1+\frac{x}{a}}}$$

Result (type 4, 211 leaves, 11 steps):

$$\frac{1}{2} x \sqrt{a^2-x^2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}-\frac{a \sqrt{a^2-x^2} \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+\frac{a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{16 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}-\frac{a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{16 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}$$

Problem 394: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{\sqrt{a^2-x^2}} \, dx$$

Optimal (type 3, 50 leaves, 1 step):

$$\frac{2 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2-x^2}}$$

Result (type 3, 50 leaves, 2 steps):

$$\frac{2 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2-x^2}}$$

Problem 395: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{\left(a^2-x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 97 leaves, 1 step):

$$\frac{x \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{a^2 \sqrt{a^2-x^2}}+\frac{\sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \text{Unintegrable}\left[\frac{x}{\left(1-\frac{x^2}{a^2}\right) \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}, x\right]}{2 a^3 \sqrt{a^2-x^2}}$$

Result (type 8, 97 leaves, 2 steps):

$$\frac{x\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{a^2\sqrt{a^2-x^2}}+\frac{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\ \text{Unintegrable}\left[\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}},x\right]}{2a^3\sqrt{a^2-x^2}}$$

Problem 398: Result optimal but 1 more steps used.

$$\int\sqrt{a^2-x^2}\ \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}dx$$

Optimal (type 4, 316 leaves, 11 steps):

$$\frac{3a\sqrt{a^2-x^2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{3x^2\sqrt{a^2-x^2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{1}{2}x\sqrt{a^2-x^2}\ \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}-\frac{a\sqrt{a^2-x^2}\ \text{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\ \text{Erf}\left[\sqrt{2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\ \text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

Result (type 4, 316 leaves, 12 steps):

$$\frac{3a\sqrt{a^2-x^2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}-\frac{3x^2\sqrt{a^2-x^2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{1}{2}x\sqrt{a^2-x^2}\ \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}-\frac{a\sqrt{a^2-x^2}\ \text{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\ \text{Erf}\left[\sqrt{2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}+\frac{3a\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\ \text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

Problem 399: Result optimal but 1 more steps used.

$$\int\frac{\text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2-x^2}}dx$$

Optimal (type 3, 50 leaves, 1 step):

$$\frac{2 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \operatorname{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2-x^2}}$$

Result (type 3, 50 leaves, 2 steps):

$$\frac{2 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \operatorname{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2-x^2}}$$

Problem 400: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{\left(a^2-x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 97 leaves, 1 step):

$$\frac{x \operatorname{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{a^2 \sqrt{a^2-x^2}}+\frac{3 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \operatorname{Unintegrable}\left[\frac{x \sqrt{\operatorname{ArcCosh}\left[\frac{x}{a}\right]}}{1-\frac{x^2}{a^2}}, x\right]}{2 a^3 \sqrt{a^2-x^2}}$$

Result (type 8, 97 leaves, 2 steps):

$$\frac{x \operatorname{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{a^2 \sqrt{a^2-x^2}}+\frac{3 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \operatorname{Unintegrable}\left[\frac{x \sqrt{\operatorname{ArcCosh}\left[\frac{x}{a}\right]}}{1-\frac{x^2}{a^2}}, x\right]}{2 a^3 \sqrt{a^2-x^2}}$$

Problem 401: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{1-x^2} \sqrt{\operatorname{ArcCosh}[x]}} \, dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$\frac{\sqrt{\pi} \sqrt{-1+x} \operatorname{Erf}\left[\sqrt{\operatorname{ArcCosh}[x]}\right]}{2 \sqrt{1-x}}+\frac{\sqrt{\pi} \sqrt{-1+x} \operatorname{Erfi}\left[\sqrt{\operatorname{ArcCosh}[x]}\right]}{2 \sqrt{1-x}}$$

Result (type 4, 83 leaves, 7 steps):

$$\frac{\sqrt{\pi}\ \sqrt{-1+x}\ \sqrt{1+x}\ \text{Erf}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2\sqrt{1-x^2}}+\frac{\sqrt{\pi}\ \sqrt{-1+x}\ \sqrt{1+x}\ \text{Erfi}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2\sqrt{1-x^2}}$$

Problem 402: Result optimal but 1 more steps used.

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 4, 438 leaves, 18 steps):

$$-\frac{5\,c^2\sqrt{c-a^2\,c\,x^2}\ \sqrt{\text{ArcCosh}[a\,x]}}{8\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}-\frac{3\,c^2\sqrt{\pi}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erf}\left[2\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+\frac{15\,c^2\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+$$
$$\frac{c^2\sqrt{\frac{\pi}{6}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erf}\left[\sqrt{6}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}-\frac{3\,c^2\sqrt{\pi}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erfi}\left[2\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+$$
$$\frac{15\,c^2\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+\frac{c^2\sqrt{\frac{\pi}{6}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erfi}\left[\sqrt{6}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}$$

Result (type 4, 438 leaves, 19 steps):

$$-\frac{5\,c^2\sqrt{c-a^2\,c\,x^2}\ \sqrt{\text{ArcCosh}[a\,x]}}{8\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}-\frac{3\,c^2\sqrt{\pi}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erf}\left[2\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+\frac{15\,c^2\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+$$
$$\frac{c^2\sqrt{\frac{\pi}{6}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erf}\left[\sqrt{6}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}-\frac{3\,c^2\sqrt{\pi}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erfi}\left[2\sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+$$
$$\frac{15\,c^2\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}+\frac{c^2\sqrt{\frac{\pi}{6}}\ \sqrt{c-a^2\,c\,x^2}\ \text{Erfi}\left[\sqrt{6}\ \sqrt{\text{ArcCosh}[a\,x]}\right]}{64\,a\sqrt{-1+a\,x}\ \sqrt{1+a\,x}}$$

Problem 403: Result optimal but 1 more steps used.

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 4, 294 leaves, 13 steps):

$$-\frac{3\,c\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{c\,\sqrt{\pi}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[2\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{32\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{c\,\sqrt{\pi}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[2\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{32\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Result (type 4, 294 leaves, 14 steps):

$$-\frac{3\,c\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{c\,\sqrt{\pi}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[2\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{32\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{c\,\sqrt{\pi}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[2\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{32\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 404: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 4, 175 leaves, 8 steps):

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Result (type 4, 175 leaves, 9 steps):

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 405: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 3, 46 leaves, 1 step):

$$\frac{2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 3, 46 leaves, 2 steps):

$$\frac{2\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\sqrt{c-a^2\,c\,x^2}}$$

Problem 406: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(c-a^2\,c\,x^2\right)^{3/2}\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(c-a^2\,c\,x^2\right)^{3/2}\sqrt{\text{ArcCosh}[a\,x]}},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$-\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Unintegrable}\left[\frac{1}{(-1+a\,x)^{3/2}\,(1+a\,x)^{3/2}\sqrt{\text{ArcCosh}[a\,x]}},\,x\right]}{c\sqrt{c-a^2\,c\,x^2}}$$

Problem 407: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(c-a^2\,c\,x^2\right)^{5/2}\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(c-a^2\,c\,x^2\right)^{5/2}\sqrt{\text{ArcCosh}[a\,x]}},\,x\right]$$

Result (type 8, 66 leaves, 1 step):

$$\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Unintegrable}\left[\frac{1}{(-1+a\,x)^{5/2}\,(1+a\,x)^{5/2}\sqrt{\text{ArcCosh}[a\,x]}},\,x\right]}{c^2\sqrt{c-a^2\,c\,x^2}}$$

Problem 410: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{\text{ArcCosh}[a\,x]^{3/2}}\,dx$$

Optimal (type 4, 170 leaves, 9 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}\,\sqrt{c-a^2 c x^2}}{a\sqrt{\text{ArcCosh}[a x]}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{a\sqrt{-1+a x}\,\sqrt{1+a x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{a\sqrt{-1+a x}\,\sqrt{1+a x}}$$

Result (type 4, 176 leaves, 10 steps):

$$\frac{2(1-a x)\sqrt{1+a x}\,\sqrt{c-a^2 c x^2}}{a\sqrt{-1+a x}\,\sqrt{\text{ArcCosh}[a x]}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{a\sqrt{-1+a x}\,\sqrt{1+a x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{a\sqrt{-1+a x}\,\sqrt{1+a x}}$$

Problem 411: Result optimal but 1 more steps used.

$$\int\frac{1}{\sqrt{c-a^2 c x^2}\,\text{ArcCosh}[a x]^{3/2}}\,dx$$

Optimal (type 3, 46 leaves, 1 step):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{a\sqrt{c-a^2 c x^2}\,\sqrt{\text{ArcCosh}[a x]}}$$

Result (type 3, 46 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{a\sqrt{c-a^2 c x^2}\,\sqrt{\text{ArcCosh}[a x]}}$$

Problem 412: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(c-a^2 c x^2\right)^{3/2}\,\text{ArcCosh}[a x]^{3/2}}\,dx$$

Optimal (type 8, 109 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{a\left(c-a^2 c x^2\right)^{3/2}\sqrt{\text{ArcCosh}[a x]}}+\frac{4 a\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^2\sqrt{\text{ArcCosh}[a x]}},x\right]}{c\sqrt{c-a^2 c x^2}}$$

Result (type 8, 120 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}}{a c(1-a x)\sqrt{1+a x}\,\sqrt{c-a^2 c x^2}\,\sqrt{\text{ArcCosh}[a x]}}+\frac{4 a\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^2\sqrt{\text{ArcCosh}[a x]}},x\right]}{c\sqrt{c-a^2 c x^2}}$$

Problem 413: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(c - a^2 c x^2\right)^{5/2} \operatorname{ArcCosh}[a x]^{3/2}} \, dx$$

Optimal (type 8, 109 leaves, 2 steps):

$$-\frac{2 \sqrt{-1 + a x} \sqrt{1 + a x}}{a \left(c - a^2 c x^2\right)^{5/2} \sqrt{\operatorname{ArcCosh}[a x]}} - \frac{8 a \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^3 \sqrt{\operatorname{ArcCosh}[a x]}}, x\right]}{c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 8, 120 leaves, 2 steps):

$$-\frac{2 \sqrt{-1 + a x}}{a c^2 (1 - a x)^2 (1 + a x)^{3/2} \sqrt{c - a^2 c x^2} \sqrt{\operatorname{ArcCosh}[a x]}} - \frac{8 a \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^3 \sqrt{\operatorname{ArcCosh}[a x]}}, x\right]}{c^2 \sqrt{c - a^2 c x^2}}$$

Problem 415: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{c - a^2 c x^2}}{\operatorname{ArcCosh}[a x]^{5/2}} \, dx$$

Optimal (type 4, 201 leaves, 7 steps):

$$-\frac{2 \sqrt{-1 + a x} \sqrt{1 + a x} \sqrt{c - a^2 c x^2}}{3 a \operatorname{ArcCosh}[a x]^{3/2}} - \frac{8 x \sqrt{c - a^2 c x^2}}{3 \sqrt{\operatorname{ArcCosh}[a x]}} + \frac{2 \sqrt{2 \pi} \sqrt{c - a^2 c x^2} \operatorname{Erf}\left[\sqrt{2} \sqrt{\operatorname{ArcCosh}[a x]}\right]}{3 a \sqrt{-1 + a x} \sqrt{1 + a x}} + \frac{2 \sqrt{2 \pi} \sqrt{c - a^2 c x^2} \operatorname{Erfi}\left[\sqrt{2} \sqrt{\operatorname{ArcCosh}[a x]}\right]}{3 a \sqrt{-1 + a x} \sqrt{1 + a x}}$$

Result (type 4, 207 leaves, 8 steps):

$$\frac{2 (1 - a x) \sqrt{1 + a x} \sqrt{c - a^2 c x^2}}{3 a \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^{3/2}} - \frac{8 x \sqrt{c - a^2 c x^2}}{3 \sqrt{\operatorname{ArcCosh}[a x]}} + \frac{2 \sqrt{2 \pi} \sqrt{c - a^2 c x^2} \operatorname{Erf}\left[\sqrt{2} \sqrt{\operatorname{ArcCosh}[a x]}\right]}{3 a \sqrt{-1 + a x} \sqrt{1 + a x}} + \frac{2 \sqrt{2 \pi} \sqrt{c - a^2 c x^2} \operatorname{Erfi}\left[\sqrt{2} \sqrt{\operatorname{ArcCosh}[a x]}\right]}{3 a \sqrt{-1 + a x} \sqrt{1 + a x}}$$

Problem 416: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c - a^2 c x^2} \operatorname{ArcCosh}[a x]^{5/2}} \, dx$$

Optimal (type 3, 48 leaves, 1 step):

$$-\frac{2\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{3\,a\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}$$

Result (type 3, 48 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{3\,a\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}$$

Problem 419: Result optimal but 1 more steps used.

$$\int x^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{8\,b\,c^3\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2^{-2\,(3+n)}\,e^{-\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2^{-2\,(3+n)}\,e^{\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 253 leaves, 7 steps):

$$-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{8\,b\,c^3\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2^{-2\,(3+n)}\,e^{-\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2^{-2\,(3+n)}\,e^{\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 420: Result optimal but 1 more steps used.

$$\int x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 379 leaves, 9 steps):

$$\frac{3^{-1-n}\,e^{-\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{e^{a/b}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3^{-1-n}\,e^{\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 379 leaves, 10 steps):

$$\frac{3^{-1-n}\,e^{-\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{e^{a/b}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3^{-1-n}\,e^{\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 421: Result optimal but 1 more steps used.

$$\int\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2^{-3-n}\,e^{-\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2^{-3-n}\,e^{\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 253 leaves, 7 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 422: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n}{x}\,dx$$

Optimal (type 8, 211 leaves, 6 steps):

$$-\frac{d\,e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,\sqrt{d-c^2 d x^2}}+$$

$$\frac{d\,e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,\sqrt{d-c^2 d x^2}}+d\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 245 leaves, 7 steps):

$$\frac{e^{-\frac{a}{b}}\,\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{e^{a/b}\,\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2 d x^2}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 423: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n}{x^2}\,dx$$

Optimal (type 8, 91 leaves, 3 steps):

$$-\frac{c\,d\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{b\,(1+n)\,\sqrt{d-c^2 d x^2}}+d\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 125 leaves, 4 steps):

$$\frac{c\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{b\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2 d x^2}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 424: Result optimal but 1 more steps used.

$$\int x^2\left(d-c^2 d x^2\right)^{3/2}\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 658 leaves, 12 steps):

$$-\frac{d\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{16\,b\,c^3\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{-\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{2^{-7-2n}\,d\,e^{-\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{2^{-7-n}\,d\,e^{-\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{2^{-7-n}\,d\,e^{\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{2^{-7-2n}\,d\,e^{\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 658 leaves, 13 steps):

$$-\frac{d\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^{1+n}}{16\,b\,c^3\,(1+n)\,\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{-\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n\left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3\,\sqrt{-1+c x}\,\sqrt{1+c x}}+$$

$$\frac{2^{-7-2n}\,d\,e^{-\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n\left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3\,\sqrt{-1+c x}\,\sqrt{1+c x}}+$$

$$\frac{2^{-7-n}\,d\,e^{-\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n\left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3\,\sqrt{-1+c x}\,\sqrt{1+c x}}-$$

$$\frac{2^{-7-n}\,d\,e^{\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n\left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3\,\sqrt{-1+c x}\,\sqrt{1+c x}}-$$

$$\frac{2^{-7-2n}\,d\,e^{\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n\left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3\,\sqrt{-1+c x}\,\sqrt{1+c x}}+$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d\,e^{\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n\left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3\,\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 425: Result optimal but 1 more steps used.

$$\int x\,(d-c^2 d x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c x])^n\,dx$$

Optimal (type 4, 578 leaves, 12 steps) :

$$-\frac{5^{-1-n}\,d\,e^{-\frac{5a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3^{-n}\,d\,e^{-\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{d\,e^{-\frac{a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{16\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{d\,e^{a/b}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{16\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3^{-n}\,d\,e^{\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5^{-1-n}\,d\,e^{\frac{5a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 578 leaves, 13 steps):

$$-\frac{5^{-1-n}\,d\,e^{-\frac{5a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3^{-n}\,d\,e^{-\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{d\,e^{-\frac{a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{16\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{d\,e^{a/b}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{16\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3^{-n}\,d\,e^{\frac{3a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5^{-1-n}\,d\,e^{\frac{5a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 426: Result optimal but 1 more steps used.

$$\int \left(d-c^2 d x^2\right)^{3/2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \, dx$$

Optimal (type 4, 450 leaves, 9 steps):

$$-\frac{3 d \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^{1+n}}{8 b c (1+n) \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2^{-2 (3+n)} d\, e^{-\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{2^{-3-n} d\, e^{-\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{2^{-3-n} d\, e^{\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{2^{-2 (3+n)} d\, e^{\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 450 leaves, 10 steps):

$$-\frac{3 d \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^{1+n}}{8 b c (1+n) \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2^{-2 (3+n)} d\, e^{-\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{2^{-3-n} d\, e^{-\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{2^{-3-n} d\, e^{\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{2^{-2 (3+n)} d\, e^{\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\, \text{ArcCosh}[c x]\right)^n \left(\frac{a+b\, \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b\, \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 427: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} \left(a+b\, \text{ArcCosh}[c x]\right)^n}{x} \, dx$$

Optimal (type 8, 414 leaves, 15 steps):

$$\frac{3^{-1-n}\, d^2\, e^{-\frac{3a}{b}} \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,\sqrt{d-c^2\,d\,x^2}} -$$

$$\frac{5\,d^2\, e^{-\frac{a}{b}} \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,\sqrt{d-c^2\,d\,x^2}} +$$

$$\frac{5\,d^2\, e^{a/b} \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,\sqrt{d-c^2\,d\,x^2}} -$$

$$\frac{3^{-1-n}\, d^2\, e^{\frac{3a}{b}} \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,\sqrt{d-c^2\,d\,x^2}} + d^2\,\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x\,\sqrt{d-c^2\,d\,x^2}},\,x\right]$$

Result (type 8, 441 leaves, 16 steps):

$$-\frac{3^{-1-n}\, d\, e^{-\frac{3a}{b}} \sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}} +$$

$$\frac{5\,d\, e^{-\frac{a}{b}} \sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}} -$$

$$\frac{5\,d\, e^{a/b} \sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}} +$$

$$\frac{3^{-1-n}\, d\, e^{\frac{3a}{b}} \sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}} - \frac{d\,\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 428: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x^2}\,dx$$

Optimal (type 8, 291 leaves, 9 steps):

$$-\frac{3\,c\,d^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}+\frac{1}{\sqrt{d-c^2\,d\,x^2}}$$

$$2^{-3-n}\,c\,d^2\,e^{-\frac{2a}{b}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]-$$

$$\frac{2^{-3-n}\,c\,d^2\,e^{\frac{2a}{b}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{d-c^2\,d\,x^2}}+$$

$$d^2\,\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x^2\,\sqrt{d-c^2\,d\,x^2}},x\right]$$

Result (type 8, 320 leaves, 10 steps):

$$\frac{3\,c\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2^{-3-n}\,c\,d\,e^{-\frac{2a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{2^{-3-n}\,c\,d\,e^{\frac{2a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{d\,\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 429: Result optimal but 1 more steps used.

$$\int x^2\left(d-c^2\,d\,x^2\right)^{5/2}\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 870 leaves, 15 steps):

$$-\frac{5 d^2 \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^{1+n}}{128 b c^3 (1+n) \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2^{-11-3 n} d^2 e^{-\frac{8 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{8 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{6 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{2^{-2 (4+n)} d^2 e^{-\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{2^{-2 (4+n)} d^2 e^{\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{6 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{2^{-11-3 n} d^2 e^{\frac{8 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{8 (a+b \text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 870 leaves, 16 steps) :

$$-\frac{5 d^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^{1+n}}{128\,b\,c^3\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2^{-11-3\,n}\,d^2\,e^{-\frac{8\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{2^{-2\,(4+n)}\,d^2\,e^{-\frac{4\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{2^{-7-n}\,d^2\,e^{-\frac{2\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-7-n}\,d^2\,e^{\frac{2\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-2\,(4+n)}\,d^2\,e^{\frac{4\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-11-3\,n}\,d^2\,e^{\frac{8\,a}{b}}\,\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 430: Result optimal but 1 more steps used.

$$\int x\,\left(d-c^2 d x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^n\,dx$$

Optimal (type 4, 793 leaves, 15 steps):

$$\frac{7^{-1-n} d^2 e^{-\frac{7 a}{b}} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{5^{-n} d^2 e^{-\frac{5 a}{b}} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{3^{1-n} d^2 e^{-\frac{3 a}{b}} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{5 d^2 e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{5 d^2 e^{a/b} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{3^{1-n} d^2 e^{\frac{3 a}{b}} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{5^{-n} d^2 e^{\frac{5 a}{b}} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{7^{-1-n} d^2 e^{\frac{7 a}{b}} \sqrt{d-c^2 d x^2} \, (a+b \text{ArcCosh}[c x])^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 793 leaves, 16 steps) :

$$\frac{7^{-1-n} d^2 e^{-\frac{7a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{5^{-n} d^2 e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{3^{1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{5 d^2 e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{5 d^2 e^{a/b} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{3^{1-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{5^{-n} d^2 e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{7^{-1-n} d^2 e^{\frac{7a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{7 (a+b \text{ArcCosh}[c x])}{b}\right]}{128 c^2 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 431: Result optimal but 1 more steps used.

$$\int \left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)^n dx$$

Optimal (type 4, 674 leaves, 12 steps):

$$-\frac{5 d^2 \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^{1+n}}{16 b c (1+n) \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{6 (a+b \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{3\times 2^{-7-2 n} d^2 e^{-\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{15\times 2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{15\times 2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{3\times 2^{-7-2 n} d^2 e^{\frac{4 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{6 (a+b \text{ArcCosh}[c x])}{b}\right]}{c \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 674 leaves, 13 steps) :

$$-\frac{5\,d^2\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{16\,b\,c\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{3\times 2^{-7-2n}\,d^2\,e^{-\frac{4a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{15\times 2^{-7-n}\,d^2\,e^{-\frac{2a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{15\times 2^{-7-n}\,d^2\,e^{\frac{2a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{3\times 2^{-7-2n}\,d^2\,e^{\frac{4a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 432: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x}\,dx$$

Optimal (type 8, 804 leaves, 27 steps):

$$-\frac{1}{32\sqrt{d-c^2 d x^2}} 5^{-1-n} d^3 e^{-\frac{5a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{5\left(a+b \operatorname{ArcCosh}[c x]\right)}{b}\right]-$$
$$\frac{1}{32\sqrt{d-c^2 d x^2}} 5\times 3^{-1-n} d^3 e^{-\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{3\left(a+b \operatorname{ArcCosh}[c x]\right)}{b}\right]+$$
$$\frac{3^{-n} d^3 e^{-\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8\sqrt{d-c^2 d x^2}}-$$
$$\frac{11 d^3 e^{-\frac{a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{16\sqrt{d-c^2 d x^2}}+$$
$$\frac{11 d^3 e^{a/b} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{16\sqrt{d-c^2 d x^2}}+$$
$$\frac{5\times 3^{-1-n} d^3 e^{\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{32\sqrt{d-c^2 d x^2}}-$$
$$\frac{3^{-n} d^3 e^{\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8\sqrt{d-c^2 d x^2}}+$$
$$\frac{5^{-1-n} d^3 e^{\frac{5a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{5(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{32\sqrt{d-c^2 d x^2}}+d^3 \operatorname{Unintegrable}\left[\frac{\left(a+b \operatorname{ArcCosh}[c x]\right)^n}{x\sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 841 leaves, 28 steps) :

$$\frac{5^{-1-n} d^2 e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{5\times 3^{-1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{3^{-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{11 d^2 e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{16 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{11 d^2 e^{a/b} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{16 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{5\times 3^{-1-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{3^{-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{5^{-1-n} d^2 e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5 (a+b \text{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{d^2 \sqrt{d-c^2 d x^2} \,\text{Unintegrable}\left[\frac{(a+b \text{ArcCosh}[c x])^n}{x \sqrt{-1+c x} \sqrt{1+c x}},x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 433: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)^n}{x^2} \, dx$$

Optimal (type 8, 485 leaves, 18 steps) :

$$-\frac{15\,c\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{8\,b\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}-\frac{1}{\sqrt{d-c^2\,d\,x^2}}$$

$$2^{-2\,(3+n)}\,c\,d^3\,e^{-\frac{4a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]+$$

$$\frac{1}{\sqrt{d-c^2\,d\,x^2}}2^{-2-n}\,c\,d^3\,e^{-\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]-$$

$$\frac{2^{-2-n}\,c\,d^3\,e^{\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{d-c^2\,d\,x^2}}+\frac{1}{\sqrt{d-c^2\,d\,x^2}}$$

$$2^{-2\,(3+n)}\,c\,d^3\,e^{\frac{4a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]+$$

$$d^3\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{d-c^2\,d\,x^2}},x\right]$$

Result (type 8, 522 leaves, 19 steps):

$$\frac{15\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{8\,b\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2^{-2\,(3+n)}\,c\,d^2\,e^{-\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-2-n}\,c\,d^2\,e^{-\frac{2a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{2^{-2-n}\,c\,d^2\,e^{\frac{2a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{2^{-2\,(3+n)}\,c\,d^2\,e^{\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{d^2\,\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 434: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 4, 323 leaves, 9 steps):

$$\frac{3^{-1-n}\, e^{-\frac{3a}{b}} \sqrt{-1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{3 (a+b\,\text{ArcCosh}[c x])}{b}\right]}{8 c^4 \sqrt{1-c x}} + \frac{3\, e^{-\frac{a}{b}} \sqrt{-1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]}{8 c^4 \sqrt{1-c x}} - \frac{3\, e^{a/b} \sqrt{-1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{a+b\,\text{ArcCosh}[c x]}{b}\right]}{8 c^4 \sqrt{1-c x}} - \frac{3^{-1-n}\, e^{\frac{3a}{b}} \sqrt{-1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{3 (a+b\,\text{ArcCosh}[c x])}{b}\right]}{8 c^4 \sqrt{1-c x}}$$

Result (type 4, 375 leaves, 10 steps):

$$\frac{3^{-1-n}\, e^{-\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{3 (a+b\,\text{ArcCosh}[c x])}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}} + \frac{3\, e^{-\frac{a}{b}} \sqrt{-1+c x} \sqrt{1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}} - \frac{3\, e^{a/b} \sqrt{-1+c x} \sqrt{1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{a+b\,\text{ArcCosh}[c x]}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}} - \frac{3^{-1-n}\, e^{\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{3 (a+b\,\text{ArcCosh}[c x])}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}}$$

Problem 435: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a+b\,\text{ArcCosh}[c x])^n}{\sqrt{1-c^2 x^2}}\, dx$$

Optimal (type 4, 211 leaves, 6 steps):

$$\frac{\sqrt{-1+c x}\, (a+b\,\text{ArcCosh}[c x])^{1+n}}{2 b c^3 (1+n) \sqrt{1-c x}} + \frac{2^{-3-n}\, e^{-\frac{2a}{b}} \sqrt{-1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, -\frac{2 (a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{1-c x}} - \frac{2^{-3-n}\, e^{\frac{2a}{b}} \sqrt{-1+c x}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n, \frac{2 (a+b\,\text{ArcCosh}[c x])}{b}\right]}{c^3 \sqrt{1-c x}}$$

Result (type 4, 250 leaves, 7 steps):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c^3\,(1+n)\,\sqrt{1-c^2\,x^2}}+\frac{2^{-3-n}\,e^{-\frac{2\,a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{1-c^2\,x^2}}-\frac{2^{-3-n}\,e^{\frac{2\,a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{1-c^2\,x^2}}$$

Problem 436: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 4, 154 leaves, 4 steps):

$$\frac{e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{1-c\,x}}-\frac{e^{a/b}\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{1-c\,x}}$$

Result (type 4, 180 leaves, 5 steps):

$$\frac{e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{1-c^2\,x^2}}-\frac{e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{1-c^2\,x^2}}$$

Problem 437: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{b\,c\,(1+n)\,\sqrt{1-c\,x}}$$

Result (type 3, 56 leaves, 2 steps):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{b\,c\,(1+n)\,\sqrt{1-c^2\,x^2}}$$

Problem 438: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x\,\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x\,\sqrt{1-c^2\,x^2}},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 439: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x^2\,\sqrt{1-c^2\,x^2}},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 440: Result optimal but 1 more steps used.

$$\int \frac{x^3 \left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 379 leaves, 9 steps):

$$\frac{3^{-1-n}\,e^{-\frac{3a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{3\,e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3\,e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3^{-1-n}\,e^{\frac{3a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 379 leaves, 10 steps):

$$\frac{3^{-1-n}\,e^{-\frac{3a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{3\,e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3\,e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{3^{-1-n}\,e^{\frac{3a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 441: Result optimal but 1 more steps used.

$$\int \frac{x^2 \left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c^3\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 253 leaves, 7 steps):

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c^3\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{c^3\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 442: Result optimal but 1 more steps used.

$$\int\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 182 leaves, 4 steps):

$$\frac{e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{e^{a/b}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 182 leaves, 5 steps):

$$\frac{e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{e^{a/b}\,\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 443: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcCosh}[c\,x])^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 57 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{b\,c\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 57 leaves, 2 steps):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{b\,c\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 444: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{d-c^2\,d\,x^2}},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 445: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{d-c^2\,d\,x^2}},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},x\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 446: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\,(a+b\,\text{ArcCosh}[c\,x])^n}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2\,(a+b\,\text{ArcCosh}[c\,x])^n}{\left(d-c^2\,d\,x^2\right)^{3/2}},x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^2\,(a+b\,\text{ArcCosh}[c\,x])^n}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 447: Result valid but suboptimal antiderivative.

$$\int\frac{x\,(a+b\,\text{ArcCosh}[c\,x])^n}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 8, 29 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x\,(a+b\,\text{ArcCosh}[c\,x])^n}{\left(d-c^2\,d\,x^2\right)^{3/2}},x\right]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x\,(a+b\,\text{ArcCosh}[c\,x])^n}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 448: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{(d-c^2\,d\,x^2)^{3/2}},\,x\right]$$

Result (type 8, 69 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},\,x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 449: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,(d-c^2\,d\,x^2)^{3/2}},\,x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},\,x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 450: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,(d-c^2\,d\,x^2)^{3/2}},\,x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},\,x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 451: Result valid but suboptimal antiderivative.

$$\int \frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^n}{\sqrt{1-c^2\,x^2}},\,x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^n}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 457: Result valid but suboptimal antiderivative.

$$\int (f\,x)^m\,\left(d-c^2\,d\,x^2\right)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^n\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[(f\,x)^m\,\left(d-c^2\,d\,x^2\right)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^n,\,x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{d\,\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\left[(f\,x)^m\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^n,\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 458: Result valid but suboptimal antiderivative.

$$\int (f\,x)^m\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[(f\,x)^m\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n,\,x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\left[(f\,x)^m\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^n,\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 459: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \text{ArcCosh}[c x])^n}{\sqrt{d - c^2 d x^2}} \, dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a + b \text{ArcCosh}[c x])^n}{\sqrt{d - c^2 d x^2}}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \, \text{Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^n}{\sqrt{-1+c x} \sqrt{1+c x}}, x\right]}{\sqrt{d - c^2 d x^2}}$$

Problem 460: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \text{ArcCosh}[c x])^n}{(d - c^2 d x^2)^{3/2}} \, dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a + b \text{ArcCosh}[c x])^n}{(d - c^2 d x^2)^{3/2}}, x\right]$$

Result (type 8, 74 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \, \text{Unintegrable}\left[\frac{(f x)^m (a+b \text{ArcCosh}[c x])^n}{(-1+c x)^{3/2} (1+c x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d x^2}}$$

Test results for the 296 problems in "7.2.5 Inverse hyperbolic cosine functions.m"

Problem 61: Unable to integrate problem.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \text{ArcCosh}[c x])}{f + g x} \, dx$$

Optimal (type 4, 1270 leaves, ? steps):

$$-\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}}{g^3}+\frac{b\,c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}}{g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^2\,d\,(c\,f-g)\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{a\,d\,\left(2+3\,c\,x-2\,c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}}{6\,g}+\frac{b\,c\,d\,x\,\left(-12-9\,c\,x+4\,c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}}{36\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{g^3}-$$

$$\frac{a\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{2\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,\left(2+3\,c\,x-2\,c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{6\,g}-\frac{b\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]^2}{4\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{c\,d\,(c\,f-g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2\,g^2}-\frac{d\,(c\,f-g)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,b\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{d\,(c\,f-g)^2\,(c\,f+g)^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+$$

$$\frac{d\,(c\,f-g)\,(c\,f+g)\,\left(1-c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}-\frac{2\,a\,d\,(c\,f-g)^{3/2}\,(c\,f+g)^{3/2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c\,f+g}\,\sqrt{1+c\,x}}{\sqrt{c\,f-g}\,\sqrt{-1+c\,x}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 8, 1150 leaves, 28 steps) :

$$\frac{b\,c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}}{g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^2\,d\,(c\,f-g)\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}{g^3\,(1-c\,x)\,(1+c\,x)}-$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{g^3}+\frac{c\,d\,(c\,f-g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2\,g^2}-$$

$$\frac{d\,(c\,f-g)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,b\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{d\,(c\,f-g)^2\,(c\,f+g)^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+\frac{d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+$$

$$\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}\Big[\frac{g+c^2\,f\,x}{\sqrt{c^2\,f^2-g^2}\,\sqrt{-1+c^2\,x^2}}\Big]}{g^4\,(1-c\,x)\,(1+c\,x)}-$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\Big[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\Big[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{c\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\big[(-1+c\,x)^{3/2}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x]),\,x\big]}{g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 144: Result valid but suboptimal antiderivative.

$$\int\frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{ArcCosh}[c+d\,x])^3}\,dx$$

Optimal (type 4, 252 leaves, 18 steps) :

$$-\frac{e^2\sqrt{-1+c+d\,x}\,\left(c+d\,x\right)^2\sqrt{1+c+d\,x}}{2\,b\,d\,\left(a+b\,\text{ArcCosh}[c+d\,x]\right)^2}+\frac{e^2\,\left(c+d\,x\right)}{b^2\,d\,\left(a+b\,\text{ArcCosh}[c+d\,x]\right)}-\frac{3\,e^2\,\left(c+d\,x\right)^3}{2\,b^2\,d\,\left(a+b\,\text{ArcCosh}[c+d\,x]\right)}-\frac{e^2\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^3\,d}-\frac{9\,e^2\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c+d\,x])}{b}\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{8\,b^3\,d}+\frac{e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c+d\,x])}{b}\right]}{8\,b^3\,d}$$

Result (type 4, 311 leaves, 18 steps):

$$-\frac{e^2\sqrt{-1+c+d\,x}\,\left(c+d\,x\right)^2\sqrt{1+c+d\,x}}{2\,b\,d\,\left(a+b\,\text{ArcCosh}[c+d\,x]\right)^2}+\frac{e^2\,\left(c+d\,x\right)}{b^2\,d\,\left(a+b\,\text{ArcCosh}[c+d\,x]\right)}-\frac{3\,e^2\,\left(c+d\,x\right)^3}{2\,b^2\,d\,\left(a+b\,\text{ArcCosh}[c+d\,x]\right)}-\frac{9\,e^2\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c+d\,x]\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^3\,d}+\frac{e^2\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{b^3\,d}-\frac{9\,e^2\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c+d\,x]\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c+d\,x]\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c+d\,x]\right]}{8\,b^3\,d}-\frac{e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]}{b^3\,d}$$

Test results for the 243 problems in "7.3.2 (d x)^m (a+b arctanh(c x^n))^p.m"

Problem 64: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^7\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2\,dx$$

Optimal (type 3, 125 leaves, 12 steps):

$$\frac{a\,b\,x^2}{4\,c^3}+\frac{b^2\,x^4}{24\,c^2}+\frac{b^2\,x^2\,\text{ArcTanh}\left[c\,x^2\right]}{4\,c^3}+\frac{b\,x^6\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{12\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{8\,c^4}+\frac{1}{8}\,x^8\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2+\frac{b^2\,\text{Log}\left[1-c^2\,x^4\right]}{6\,c^4}$$

Result (type 4, 636 leaves, 62 steps):

$$\frac{a\,b\,x^2}{8\,c^3}+\frac{23\,b^2\,x^2}{192\,c^3}+\frac{b^2\,x^4}{128\,c^2}-\frac{7\,b^2\,x^6}{576\,c}-\frac{b^2\,x^8}{256}+\frac{3\,b^2\,\left(1-c\,x^2\right)^2}{32\,c^4}-\frac{b^2\,\left(1-c\,x^2\right)^3}{36\,c^4}+\frac{b^2\,\left(1-c\,x^2\right)^4}{256\,c^4}-\frac{5\,b^2\,\mathrm{Log}\left[1-c\,x^2\right]}{192\,c^4}+$$
$$\frac{b^2\,\left(1-c\,x^2\right)\,\mathrm{Log}\left[1-c\,x^2\right]}{16\,c^4}+\frac{b^2\,\mathrm{Log}\left[1-c\,x^2\right]^2}{32\,c^4}-\frac{b\,x^4\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)}{32\,c^2}+\frac{b\,x^6\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)}{48\,c}-\frac{1}{64}\,b\,x^8\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)+$$
$$\frac{1}{32}\,x^8\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)^2-\frac{1}{192}\,b\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)\,\left(\frac{48\,\left(1-c\,x^2\right)}{c^4}-\frac{36\,\left(1-c\,x^2\right)^2}{c^4}+\frac{16\,\left(1-c\,x^2\right)^3}{c^4}-\frac{3\,\left(1-c\,x^2\right)^4}{c^4}-\frac{12\,\mathrm{Log}\left[1-c\,x^2\right]}{c^4}\right)-$$
$$\frac{b\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)\,\mathrm{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{16\,c^4}+\frac{b^2\,\mathrm{Log}\left[1+c\,x^2\right]}{24\,c^4}+\frac{b^2\,x^6\,\mathrm{Log}\left[1+c\,x^2\right]}{24\,c}+\frac{b^2\,\left(1+c\,x^2\right)\,\mathrm{Log}\left[1+c\,x^2\right]}{8\,c^4}+\frac{b^2\,\mathrm{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\mathrm{Log}\left[1+c\,x^2\right]}{16\,c^4}+$$
$$\frac{1}{16}\,b\,x^8\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)\,\mathrm{Log}\left[1+c\,x^2\right]-\frac{b^2\,\mathrm{Log}\left[1+c\,x^2\right]^2}{32\,c^4}+\frac{1}{32}\,b^2\,x^8\,\mathrm{Log}\left[1+c\,x^2\right]^2+\frac{b^2\,\mathrm{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]}{16\,c^4}+\frac{b^2\,\mathrm{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{16\,c^4}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x^5\,\left(a+b\,\mathrm{ArcTanh}\left[c\,x^2\right]\right)^2\,dx$$

Optimal (type 4, 146 leaves, 10 steps):

$$\frac{b^2\,x^2}{6\,c^2}-\frac{b^2\,\mathrm{ArcTanh}\left[c\,x^2\right]}{6\,c^3}+\frac{b\,x^4\,\left(a+b\,\mathrm{ArcTanh}\left[c\,x^2\right]\right)}{6\,c}+\frac{\left(a+b\,\mathrm{ArcTanh}\left[c\,x^2\right]\right)^2}{6\,c^3}+$$
$$\frac{1}{6}\,x^6\,\left(a+b\,\mathrm{ArcTanh}\left[c\,x^2\right]\right)^2-\frac{b\,\left(a+b\,\mathrm{ArcTanh}\left[c\,x^2\right]\right)\,\mathrm{Log}\left[\frac{2}{1-c\,x^2}\right]}{3\,c^3}-\frac{b^2\,\mathrm{PolyLog}\left[2,\,1-\frac{2}{1-c\,x^2}\right]}{6\,c^3}$$

Result (type 4, 536 leaves, 53 steps):

$$-\frac{a\,b\,x^2}{6\,c^2}+\frac{19\,b^2\,x^2}{72\,c^2}-\frac{5\,b^2\,x^4}{144\,c}-\frac{b^2\,x^6}{108}+\frac{b^2\,\left(1-c\,x^2\right)^2}{16\,c^3}-\frac{b^2\,\left(1-c\,x^2\right)^3}{108\,c^3}+\frac{b^2\,\mathrm{Log}\left[1-c\,x^2\right]}{72\,c^3}-\frac{b^2\,\left(1-c\,x^2\right)\,\mathrm{Log}\left[1-c\,x^2\right]}{12\,c^3}+$$
$$\frac{b^2\,\mathrm{Log}\left[1-c\,x^2\right]^2}{24\,c^3}+\frac{b\,x^4\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)}{24\,c}-\frac{1}{36}\,b\,x^6\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)+\frac{1}{24}\,x^6\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)^2-$$
$$\frac{1}{72}\,b\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)\,\left(\frac{18\,\left(1-c\,x^2\right)}{c^3}-\frac{9\,\left(1-c\,x^2\right)^2}{c^3}+\frac{2\,\left(1-c\,x^2\right)^3}{c^3}-\frac{6\,\mathrm{Log}\left[1-c\,x^2\right]}{c^3}\right)+\frac{b\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)\,\mathrm{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{12\,c^3}-$$
$$\frac{b^2\,\mathrm{Log}\left[1+c\,x^2\right]}{12\,c^3}+\frac{b^2\,x^4\,\mathrm{Log}\left[1+c\,x^2\right]}{12\,c}+\frac{b^2\,\mathrm{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\mathrm{Log}\left[1+c\,x^2\right]}{12\,c^3}+\frac{1}{12}\,b\,x^6\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)\,\mathrm{Log}\left[1+c\,x^2\right]+$$
$$\frac{b^2\,\mathrm{Log}\left[1+c\,x^2\right]^2}{24\,c^3}+\frac{1}{24}\,b^2\,x^6\,\mathrm{Log}\left[1+c\,x^2\right]^2-\frac{b^2\,\mathrm{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]}{12\,c^3}+\frac{b^2\,\mathrm{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{12\,c^3}$$

Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2 dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{a\,b\,x^2}{2\,c}+\frac{b^2\,x^2\,\text{ArcTanh}\left[c\,x^2\right]}{2\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{4\,c^2}+\frac{1}{4}\,x^4\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2+\frac{b^2\,\text{Log}\left[1-c^2\,x^4\right]}{4\,c^2}$$

Result (type 4, 524 leaves, 44 steps):

$$\frac{3\,a\,b\,x^2}{4\,c}-\frac{b^2\,x^4}{16}+\frac{b^2\left(1-c\,x^2\right)^2}{32\,c^2}+\frac{b^2\left(1+c\,x^2\right)^2}{32\,c^2}-\frac{b^2\,\text{Log}\left[1-c\,x^2\right]}{16\,c^2}+\frac{3\,b^2\left(1-c\,x^2\right)\text{Log}\left[1-c\,x^2\right]}{8\,c^2}-\frac{1}{16}\,b\,x^4\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)+$$
$$\frac{b\left(1-c\,x^2\right)^2\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{16\,c^2}-\frac{\left(1-c\,x^2\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{8\,c^2}+\frac{\left(1-c\,x^2\right)^2\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{16\,c^2}-$$
$$\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^2\right)\right]}{8\,c^2}-\frac{b^2\,\text{Log}\left[1+c\,x^2\right]}{16\,c^2}+\frac{1}{16}\,b^2\,x^4\,\text{Log}\left[1+c\,x^2\right]+\frac{3\,b^2\left(1+c\,x^2\right)\text{Log}\left[1+c\,x^2\right]}{8\,c^2}-$$
$$\frac{b^2\left(1+c\,x^2\right)^2\text{Log}\left[1+c\,x^2\right]}{16\,c^2}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^2\right)\right]\text{Log}\left[1+c\,x^2\right]}{8\,c^2}+\frac{1}{8}\,b\,x^4\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\text{Log}\left[1+c\,x^2\right]-$$
$$\frac{b^2\left(1+c\,x^2\right)\text{Log}\left[1+c\,x^2\right]^2}{8\,c^2}+\frac{b^2\left(1+c\,x^2\right)^2\text{Log}\left[1+c\,x^2\right]^2}{16\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^2\right)\right]}{8\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^2\right)\right]}{8\,c^2}$$

Problem 67: Result valid but suboptimal antiderivative.

$$\int x \left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2 dx$$

Optimal (type 4, 94 leaves, 6 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{2\,c}+\frac{1}{2}\,x^2\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2-\frac{b\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\text{Log}\left[\frac{2}{1-c\,x^2}\right]}{c}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x^2}\right]}{2\,c}$$

Result (type 4, 207 leaves, 28 steps):

$$-\frac{\left(1-c\,x^2\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{8\,c}+\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^2\right)\right]}{4\,c}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^2\right)\right]\text{Log}\left[1+c\,x^2\right]}{4\,c}+$$
$$\frac{1}{4}\,b\,x^2\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\text{Log}\left[1+c\,x^2\right]+\frac{b^2\left(1+c\,x^2\right)\text{Log}\left[1+c\,x^2\right]^2}{8\,c}-\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^2\right)\right]}{4\,c}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^2\right)\right]}{4\,c}$$

Problem 69: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x^2])^2}{x^3}\,dx$$

Optimal (type 4, 87 leaves, 5 steps):

$$\frac{1}{2}c\left(a+b\,\text{ArcTanh}[c\,x^2]\right)^2-\frac{(a+b\,\text{ArcTanh}[c\,x^2])^2}{2\,x^2}+b\,c\left(a+b\,\text{ArcTanh}[c\,x^2]\right)\text{Log}\left[2-\frac{2}{1+c\,x^2}\right]-\frac{1}{2}b^2\,c\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x^2}\right]$$

Result (type 4, 237 leaves, 24 steps):

$$2\,a\,b\,c\,\text{Log}[x]-\frac{(1-c\,x^2)\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)^2}{8\,x^2}-\frac{1}{4}b\,c\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^2\right)\right]-\frac{1}{4}b^2\,c\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^2\right)\right]\text{Log}[1+c\,x^2]-\frac{b\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)\text{Log}[1+c\,x^2]}{4\,x^2}-\frac{b^2\left(1+c\,x^2\right)\text{Log}[1+c\,x^2]^2}{8\,x^2}-\frac{1}{2}b^2\,c\,\text{PolyLog}[2,-c\,x^2]+\frac{1}{2}b^2\,c\,\text{PolyLog}[2,c\,x^2]+\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^2\right)\right]-\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^2\right)\right]$$

Problem 70: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x^2])^2}{x^5}\,dx$$

Optimal (type 3, 88 leaves, 9 steps):

$$-\frac{b\,c\left(a+b\,\text{ArcTanh}[c\,x^2]\right)}{2\,x^2}+\frac{1}{4}c^2\left(a+b\,\text{ArcTanh}[c\,x^2]\right)^2-\frac{(a+b\,\text{ArcTanh}[c\,x^2])^2}{4\,x^4}+b^2\,c^2\,\text{Log}[x]-\frac{1}{4}b^2\,c^2\,\text{Log}[1-c^2\,x^4]$$

Result (type 4, 360 leaves, 46 steps):

$$b^2\,c^2\,\text{Log}[x]-\frac{1}{8}b^2\,c^2\,\text{Log}[1-c\,x^2]-\frac{b\,c\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)}{8\,x^2}-\frac{b\,c\left(1-c\,x^2\right)\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)}{8\,x^2}+\frac{1}{16}c^2\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)^2-\frac{(2\,a-b\,\text{Log}[1-c\,x^2])^2}{16\,x^4}+\frac{1}{8}b\,c^2\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^2\right)\right]-\frac{1}{4}b^2\,c^2\,\text{Log}[1+c\,x^2]-\frac{b^2\,c\,\text{Log}[1+c\,x^2]}{4\,x^2}-\frac{1}{8}b^2\,c^2\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^2\right)\right]\text{Log}[1+c\,x^2]-\frac{b\left(2\,a-b\,\text{Log}[1-c\,x^2]\right)\text{Log}[1+c\,x^2]}{8\,x^4}+\frac{1}{16}b^2\,c^2\,\text{Log}[1+c\,x^2]^2-\frac{b^2\,\text{Log}[1+c\,x^2]^2}{16\,x^4}-\frac{1}{8}b^2\,c^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^2\right)\right]-\frac{1}{8}b^2\,c^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^2\right)\right]$$

Problem 77: Result valid but suboptimal antiderivative.

$$\int x^3 \left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^3 \, dx$$

Optimal (type 4, 141 leaves, 9 steps):

$$\frac{3\,b\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{4\,c^2}+\frac{3\,b\,x^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{4\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^3}{4\,c^2}+\frac{1}{4}\,x^4\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^3-\frac{3\,b^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,\text{Log}\left[\frac{2}{1-c\,x^2}\right]}{2\,c^2}-\frac{3\,b^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x^2}\right]}{4\,c^2}$$

Result (type 4, 479 leaves, 155 steps):

$$-\frac{3\,b\,\left(1-c\,x^2\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{16\,c^2}-\frac{\left(1-c\,x^2\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^3}{16\,c^2}+\frac{\left(1-c\,x^2\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^3}{32\,c^2}+\frac{3\,b^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{8\,c^2}+\frac{3\,b^3\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\text{Log}\left[1+c\,x^2\right]}{8\,c^2}+\frac{3\,b^2\,x^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[1+c\,x^2\right]}{8\,c}-\frac{3\,b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2\,\text{Log}\left[1+c\,x^2\right]}{32\,c^2}+\frac{3}{32}\,b\,x^4\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2\,\text{Log}\left[1+c\,x^2\right]+\frac{3\,b^3\,\left(1+c\,x^2\right)\,\text{Log}\left[1+c\,x^2\right]^2}{16\,c^2}-\frac{3\,b^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[1+c\,x^2\right]^2}{32\,c^2}+\frac{3}{32}\,b^2\,x^4\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[1+c\,x^2\right]^2-\frac{b^3\,\left(1+c\,x^2\right)\,\text{Log}\left[1+c\,x^2\right]^3}{16\,c^2}+\frac{b^3\,\left(1+c\,x^2\right)^2\,\text{Log}\left[1+c\,x^2\right]^3}{32\,c^2}-\frac{3\,b^3\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]}{8\,c^2}+\frac{3\,b^3\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{8\,c^2}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int x \left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^3 \, dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^3}{2\,c}+\frac{1}{2}\,x^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^3-\frac{3\,b\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2\,\text{Log}\left[\frac{2}{1-c\,x^2}\right]}{2\,c}-\frac{3\,b^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x^2}\right]}{2\,c}+\frac{3\,b^3\,\text{PolyLog}\left[3,\,1-\frac{2}{1-c\,x^2}\right]}{4\,c}$$

Result (type 4, 390 leaves, 82 steps):

$$-\frac{\left(1-c x^2\right) \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^3}{16 c}+\frac{3 b \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[\frac{1}{2} \left(1+c x^2\right)\right]}{8 c}-\frac{3 b \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]}{16 c}+$$
$$\frac{3}{16} b x^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]+\frac{3 b^3 \operatorname{Log}\left[\frac{1}{2} \left(1-c x^2\right)\right] \operatorname{Log}\left[1+c x^2\right]^2}{8 c}+\frac{3 b^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2}{16 c}+$$
$$\frac{3}{16} b^2 x^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2+\frac{b^3 \left(1+c x^2\right) \operatorname{Log}\left[1+c x^2\right]^3}{16 c}-\frac{3 b^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1-c x^2\right)\right]}{4 c}+$$
$$\frac{3 b^3 \operatorname{Log}\left[1+c x^2\right] \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1+c x^2\right)\right]}{4 c}-\frac{3 b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} \left(1-c x^2\right)\right]}{4 c}-\frac{3 b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} \left(1+c x^2\right)\right]}{4 c}$$

Problem 80: Unable to integrate problem.

$$\int \frac{\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3}{x^3} \, dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\frac{1}{2} c \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3-\frac{\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3}{2 x^2}+\frac{3}{2} b c \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^2 \operatorname{Log}\left[2-\frac{2}{1+c x^2}\right]-$$
$$\frac{3}{2} b^2 c \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right) \operatorname{PolyLog}\left[2,-1+\frac{2}{1+c x^2}\right]-\frac{3}{4} b^3 c \operatorname{PolyLog}\left[3,-1+\frac{2}{1+c x^2}\right]$$

Result (type 8, 284 leaves, 16 steps):

$$\frac{3}{16} b c \operatorname{Log}\left[c x^2\right] \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2-\frac{\left(1-c x^2\right) \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^3}{16 x^2}+$$
$$\frac{3}{16} b^3 c \operatorname{Log}\left[-c x^2\right] \operatorname{Log}\left[1+c x^2\right]^2-\frac{b^3 \left(1+c x^2\right) \operatorname{Log}\left[1+c x^2\right]^3}{16 x^2}-\frac{3}{8} b^2 c \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{PolyLog}\left[2, 1-c x^2\right]+$$
$$\frac{3}{8} b^3 c \operatorname{Log}\left[1+c x^2\right] \operatorname{PolyLog}\left[2, 1+c x^2\right]-\frac{3}{8} b^3 c \operatorname{PolyLog}\left[3, 1-c x^2\right]-\frac{3}{8} b^3 c \operatorname{PolyLog}\left[3, 1+c x^2\right]+$$
$$\frac{3}{8} b \operatorname{Unintegrable}\left[\frac{\left(-2 a+b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]}{x^3}, x\right]-\frac{3}{8} b^2 \operatorname{Unintegrable}\left[\frac{\left(-2 a+b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2}{x^3}, x\right]$$

Problem 81: Unable to integrate problem.

$$\int \frac{\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3}{x^5} \, dx$$

Optimal (type 4, 139 leaves, 8 steps):

$$\frac{3}{4} b c^2 \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^2-\frac{3 b c \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^2}{4 x^2}+\frac{1}{4} c^2 \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3-\frac{\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3}{4 x^4}+\frac{3}{2} b^2 c^2 \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right) \operatorname{Log}\left[2-\frac{2}{1+c x^2}\right]-\frac{3}{4} b^3 c^2 \operatorname{PolyLog}\left[2,-1+\frac{2}{1+c x^2}\right]$$

Result (type 8, 437 leaves, 29 steps):

$$\frac{3}{4} a b^2 c^2 \operatorname{Log}[x]-\frac{3 b c \left(1-c x^2\right) \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2}{32 x^2}+\frac{3}{32} b c^2 \operatorname{Log}\left[c x^2\right] \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2+\frac{1}{32} c^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^3-\frac{\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^3}{32 x^4}-\frac{3 b^3 c \left(1+c x^2\right) \operatorname{Log}\left[1+c x^2\right]^2}{32 x^2}-\frac{3}{32} b^3 c^2 \operatorname{Log}\left[-c x^2\right] \operatorname{Log}\left[1+c x^2\right]^2+\frac{1}{32} b^3 c^2 \operatorname{Log}\left[1+c x^2\right]^3-\frac{b^3 \operatorname{Log}\left[1+c x^2\right]^3}{32 x^4}-\frac{3}{16} b^3 c^2 \operatorname{PolyLog}\left[2,-c x^2\right]+\frac{3}{16} b^3 c^2 \operatorname{PolyLog}\left[2,c x^2\right]-\frac{3}{16} b^2 c^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{PolyLog}\left[2,1-c x^2\right]-\frac{3}{16} b^3 c^2 \operatorname{Log}\left[1+c x^2\right] \operatorname{PolyLog}\left[2,1+c x^2\right]-\frac{3}{16} b^3 c^2 \operatorname{PolyLog}\left[3,1-c x^2\right]+\frac{3}{16} b^3 c^2 \operatorname{PolyLog}\left[3,1+c x^2\right]+\frac{3}{8} b \operatorname{Unintegrable}\left[\frac{\left(-2 a+b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]}{x^5},x\right]-\frac{3}{8} b^2 \operatorname{Unintegrable}\left[\frac{\left(-2 a+b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2}{x^5},x\right]$$

Problem 82: Result optimal but 1 more steps used.

$$\int (d x)^{5/2} \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right) dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\frac{8 b d (d x)^{3/2}}{21 c}+\frac{2 b d^{5/2} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}+\frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1-\frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}-\frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1+\frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}+\frac{2 (d x)^{7/2} \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)}{7 d}-\frac{2 b d^{5/2} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}-\frac{b d^{5/2} \operatorname{Log}\left[\sqrt{d}+\sqrt{c} \sqrt{d} x-\sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}}+\frac{b d^{5/2} \operatorname{Log}\left[\sqrt{d}+\sqrt{c} \sqrt{d} x+\sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}}$$

Result (type 3, 317 leaves, 17 steps):

$$\frac{8 b d (d x)^{3/2}}{21 c}+\frac{2 b d^{5/2} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}+\frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1-\frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}-\frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1+\frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}+\frac{2 (d x)^{7/2} \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)}{7 d}-\frac{2 b d^{5/2} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}}-\frac{b d^{5/2} \operatorname{Log}\left[\sqrt{d}+\sqrt{c} \sqrt{d} x-\sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}}+\frac{b d^{5/2} \operatorname{Log}\left[\sqrt{d}+\sqrt{c} \sqrt{d} x+\sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}}$$

Problem 83: Result optimal but 1 more steps used.

$$\int (d\,x)^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\frac{8\,b\,d\,\sqrt{d\,x}}{5\,c}-\frac{2\,b\,d^{3/2}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}-\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{2\,(d\,x)^{5/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{5\,d}-\frac{2\,b\,d^{3/2}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}-\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}$$

Result (type 3, 317 leaves, 17 steps):

$$\frac{8\,b\,d\,\sqrt{d\,x}}{5\,c}-\frac{2\,b\,d^{3/2}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}-\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{2\,(d\,x)^{5/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{5\,d}-\frac{2\,b\,d^{3/2}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}-\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}$$

Problem 84: Result optimal but 1 more steps used.

$$\int \sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,dx$$

Optimal (type 3, 301 leaves, 15 steps):

$$\frac{2\,b\,\sqrt{d}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}-\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{2\,(d\,x)^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,d}-\frac{2\,b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}-\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}$$

Result (type 3, 301 leaves, 16 steps):

$$\frac{2\,b\,\sqrt{d}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}-\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{2\,(d\,x)^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,d}-\frac{2\,b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}-\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}$$

Problem 85: Result optimal but 1 more steps used.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{\sqrt{d\,x}}\,dx$$

Optimal (type 3, 285 leaves, 15 steps):

$$-\frac{2\,b\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{2\sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{d}-\frac{2\,b\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}+\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}$$

Result (type 3, 285 leaves, 16 steps):

$$-\frac{2\,b\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{2\sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{d}-\frac{2\,b\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}+\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}$$

Problem 86: Result optimal but 1 more steps used.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d\,x)^{3/2}}\,dx$$

Optimal (type 3, 285 leaves, 15 steps):

$$-\frac{2\,b\,c^{1/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{d\sqrt{d\,x}}+\frac{2\,b\,c^{1/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}-\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}$$

Result (type 3, 285 leaves, 16 steps):

$$-\frac{2\,b\,c^{1/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{2\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{d\sqrt{d\,x}}+$$
$$\frac{2\,b\,c^{1/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}-\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}$$

Problem 87: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d\,x)^{5/2}}\,dx$$

Optimal (type 3, 301 leaves, 15 steps):

$$\frac{2\,b\,c^{3/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}+\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{2\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,d\,(d\,x)^{3/2}}+$$
$$\frac{2\,b\,c^{3/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}+\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}$$

Result (type 3, 301 leaves, 16 steps):

$$\frac{2\,b\,c^{3/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}+\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{2\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,d\,(d\,x)^{3/2}}+$$
$$\frac{2\,b\,c^{3/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}+\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}$$

Problem 88: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d\,x)^{7/2}}\,dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$-\frac{8bc}{5d^3\sqrt{dx}}-\frac{2bc^{5/4}\text{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}+\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}-\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}-\frac{2\left(a+b\,\text{ArcTanh}\left[cx^2\right]\right)}{5d\,(dx)^{5/2}}+\frac{2bc^{5/4}\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}-\frac{bc^{5/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}\,d^{7/2}}+\frac{bc^{5/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}\,d^{7/2}}$$

Result (type 3, 317 leaves, 17 steps):

$$-\frac{8bc}{5d^3\sqrt{dx}}-\frac{2bc^{5/4}\text{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}+\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}-\frac{\sqrt{2}\,bc^{5/4}\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}-\frac{2\left(a+b\,\text{ArcTanh}\left[cx^2\right]\right)}{5d\,(dx)^{5/2}}+\frac{2bc^{5/4}\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}}-\frac{bc^{5/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}\,d^{7/2}}+\frac{bc^{5/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}\,d^{7/2}}$$

Problem 89: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}\left[cx^2\right]}{(dx)^{9/2}}\,dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$-\frac{8bc}{21d^3\,(dx)^{3/2}}+\frac{2bc^{7/4}\text{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}+\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}-\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}-\frac{2\left(a+b\,\text{ArcTanh}\left[cx^2\right]\right)}{7d\,(dx)^{7/2}}+\frac{2bc^{7/4}\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}+\frac{bc^{7/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}\,d^{9/2}}-\frac{bc^{7/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}\,d^{9/2}}$$

Result (type 3, 317 leaves, 17 steps):

$$-\frac{8bc}{21d^3\,(dx)^{3/2}}+\frac{2bc^{7/4}\text{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}+\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}-\frac{\sqrt{2}\,bc^{7/4}\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}-\frac{2\left(a+b\,\text{ArcTanh}\left[cx^2\right]\right)}{7d\,(dx)^{7/2}}+\frac{2bc^{7/4}\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}}+\frac{bc^{7/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}\,d^{9/2}}-\frac{bc^{7/4}\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}\,d^{9/2}}$$

Problem 90: Unable to integrate problem.

$$\int \sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2\,dx$$

Optimal (type 4, 6327 leaves, 238 steps):

$$-\frac{8}{9}\,a\,b\,x\,\sqrt{d\,x}-\frac{2\,\sqrt{2}\,a\,b\,\sqrt{d\,x}\,\text{ArcTan}\left[1-\sqrt{2}\,c^{1/4}\,\sqrt{x}\right]}{3\,c^{3/4}\,\sqrt{x}}+\frac{2\,\sqrt{2}\,a\,b\,\sqrt{d\,x}\,\text{ArcTan}\left[1+\sqrt{2}\,c^{1/4}\,\sqrt{x}\right]}{3\,c^{3/4}\,\sqrt{x}}-$$

$$\frac{2\,i\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]^2}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{2\,i\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]^2}{3\,c^{3/4}\,\sqrt{x}}-\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]^2}{3\,(-c)^{3/4}\,\sqrt{x}}-$$

$$\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]^2}{3\,c^{3/4}\,\sqrt{x}}+\frac{4\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{4\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-$$

$$\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{4\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-$$

$$\frac{4\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-$$

$$\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{2\,b^2\,\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+$$

$$\frac{4 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}-\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-\mathrm{i}\, c^{1/4}\right) \left(1-\mathrm{i}\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3 (-c)^{3/4} \sqrt{x}}+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 (-c)^{3/4} \sqrt{x}}+\frac{4 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-\mathrm{i}\, c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}-$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\mathrm{i} \sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1-\mathrm{i}\, c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}-\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\mathrm{i} \sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1-\mathrm{i}\, c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}-$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(\mathrm{i}\, (-c)^{1/4}-c^{1/4}\right) \left(1-\mathrm{i}\, c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}-\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(\mathrm{i}\, (-c)^{1/4}+c^{1/4}\right) \left(1-\mathrm{i}\, c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1+\mathrm{i}) \left(1-c^{1/4} \sqrt{x}\right)}{1-\mathrm{i}\, c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}-\frac{4 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1+\mathrm{i}\, c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}-$$

$$\frac{4 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}+\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}-\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}-$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}+\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}-\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+\mathrm{i}\, c^{1/4}\right) \left(1-\mathrm{i}\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3 (-c)^{3/4} \sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[\frac{(1-i)\,\left(1+c^{1/4}\,\sqrt{x}\right)}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{3\,c^{3/4}\,\sqrt{x}}+$$

$$\frac{\sqrt{2}\;a\,b\,\sqrt{d\,x}\;\mathrm{Log}\left[1-\sqrt{2}\;c^{1/4}\,\sqrt{x}+\sqrt{c}\;x\right]}{3\,c^{3/4}\,\sqrt{x}}-\frac{\sqrt{2}\;a\,b\,\sqrt{d\,x}\;\mathrm{Log}\left[1+\sqrt{2}\;c^{1/4}\,\sqrt{x}+\sqrt{c}\;x\right]}{3\,c^{3/4}\,\sqrt{x}}+\frac{4}{9}\,b^2\,x\,\sqrt{d\,x}\;\mathrm{Log}\left[1-c\,x^2\right]+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[1-c\,x^2\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[1-c\,x^2\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{4}{9}\,b\,x\,\sqrt{d\,x}\;\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)+$$

$$\frac{2\,b\,\sqrt{d\,x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)}{3\,c^{3/4}\,\sqrt{x}}-\frac{2\,b\,\sqrt{d\,x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)}{3\,c^{3/4}\,\sqrt{x}}+\frac{1}{6}\,x\,\sqrt{d\,x}\;\left(2\,a-b\,\mathrm{Log}\left[1-c\,x^2\right]\right)^2+$$

$$\frac{2}{3}\,a\,b\,x\,\sqrt{d\,x}\;\mathrm{Log}\left[1+c\,x^2\right]-\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[1+c\,x^2\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[1+c\,x^2\right]}{3\,c^{3/4}\,\sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[1+c\,x^2\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\mathrm{Log}\left[1+c\,x^2\right]}{3\,c^{3/4}\,\sqrt{x}}-\frac{1}{3}\,b^2\,x\,\sqrt{d\,x}\;\mathrm{Log}\left[1-c\,x^2\right]\,\mathrm{Log}\left[1+c\,x^2\right]+$$

$$\frac{1}{6}\,b^2\,x\,\sqrt{d\,x}\;\mathrm{Log}\left[1+c\,x^2\right]^2+\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{2}{1-(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{2\,i\,b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{2}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+$$

$$\frac{i\,b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}\;-(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{i\,b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}\;+(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-$$

$$\frac{i\,b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{(1+i)\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{2\,i\,b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{2}{1+i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{2\,b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{2}{1+(-c)^{1/4}\,\sqrt{x}}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+$$

$$\frac{b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\;-(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}+\frac{b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\;+(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-$$

$$\frac{b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\;-(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-\frac{b^2\,\sqrt{d\,x}\;\mathrm{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\;+(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\,\sqrt{x}}-$$

$$\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{(1-i)\left(1+(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}+\frac{2\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1-c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}+\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-i\, c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-\frac{2\, i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1-i\, c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}+\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\, c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i\, c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\, c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i\, c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\, c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}-c^{1/4}\right)\left(1-i\, c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\, c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}+c^{1/4}\right)\left(1-i\, c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}-\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{(1+i)\left(1-c^{1/4}\sqrt{x}\right)}{1-i\, c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}-\frac{2\, i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1+i\, c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{2\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1+c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}-\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\, c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}-\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\, c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\, c^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\, c^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}-\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\, c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\, c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+i\, c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-\frac{i\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{(1-i)\left(1+c^{1/4}\sqrt{x}\right)}{1-i\, c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\sqrt{d\, x}\,\left(a+b\,\text{ArcTanh}\left[c\, x^2\right]\right)^2, x\right]$$

Problem 91: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{\sqrt{d\,x}}\,dx$$

Optimal (type 4, 6177 leaves, 241 steps):

$$\frac{2\,a^2\,x}{\sqrt{d\,x}}-\frac{2\,\sqrt{2}\,a\,b\,\sqrt{x}\,\text{ArcTan}\left[1-\sqrt{2}\,c^{1/4}\,\sqrt{x}\right]}{c^{1/4}\,\sqrt{d\,x}}+\frac{2\,\sqrt{2}\,a\,b\,\sqrt{x}\,\text{ArcTan}\left[1+\sqrt{2}\,c^{1/4}\,\sqrt{x}\right]}{c^{1/4}\,\sqrt{d\,x}}+\frac{2\,i\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]^2}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{4\,a\,b\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]}{c^{1/4}\,\sqrt{d\,x}}+\frac{2\,i\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]^2}{c^{1/4}\,\sqrt{d\,x}}-\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]^2}{(-c)^{1/4}\,\sqrt{d\,x}}-\frac{4\,a\,b\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]^2}{c^{1/4}\,\sqrt{d\,x}}+\frac{4\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-\frac{4\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{4\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{4\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{4 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2 b^2 \sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}-\frac{4 b^2 \sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}+$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2 b^2 \sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}-c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2 b^2 \sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}+c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\left(1-c^{1/4}\sqrt{x}\right)}{1-i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{4 b^2 \sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}-\frac{4 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}+$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+$$

$$\frac{2 b^2 \sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+\frac{2 b^2 \sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}-$$

$$\frac{2 b^2 \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}}-\frac{\sqrt{2}\, a b \sqrt{x}\, \text{Log}\left[1-\sqrt{2}\, c^{1/4} \sqrt{x}+\sqrt{c}\, x\right]}{c^{1/4} \sqrt{d x}}+\frac{\sqrt{2}\, a b \sqrt{x}\, \text{Log}\left[1+\sqrt{2}\, c^{1/4} \sqrt{x}+\sqrt{c}\, x\right]}{c^{1/4} \sqrt{d x}}-$$

$$\frac{2 a b x\, \text{Log}\left[1-c x^2\right]}{\sqrt{d x}}-\frac{2 b^2 \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1-c x^2\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1-c x^2\right]}{c^{1/4} \sqrt{d x}}-$$

$$\frac{2 b^2 \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1-c x^2\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1-c x^2\right]}{c^{1/4} \sqrt{d x}}+\frac{b^2 x\, \text{Log}\left[1-c x^2\right]^2}{2 \sqrt{d x}}+\frac{2 a b x\, \text{Log}\left[1+c x^2\right]}{\sqrt{d x}}+$$

$$\frac{2 b^2 \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{(-c)^{1/4} \sqrt{d x}}-\frac{2 b^2 \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{c^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{(-c)^{1/4} \sqrt{d x}}-$$

$$\frac{2 b^2 \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{c^{1/4} \sqrt{d x}}-\frac{b^2 x\, \text{Log}\left[1-c x^2\right] \text{Log}\left[1+c x^2\right]}{\sqrt{d x}}+\frac{b^2 x\, \text{Log}\left[1+c x^2\right]^2}{2 \sqrt{d x}}+\frac{2 b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+$$

$$\frac{2 i b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1-i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}-\frac{i b^2 \sqrt{x}\, \text{PolyLog}\left[2,1+\frac{2 (-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}}-$$

$$\frac{i b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2 (-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{i b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+$$

$$\frac{2 i b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1+i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,1+\frac{2 (-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}}+$$

$$\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2 (-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}}-\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,1+\frac{2 (-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}}-$$

$$\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2 (-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{i b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{(1-i)\left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}}-$$

$$\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}-\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+\frac{2\,i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2}{1-i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1+\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}\ -c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}\ +c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1+\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}-c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}+c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+$$

$$\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{(1+i)\left(1-c^{1/4}\sqrt{x}\right)}{1-i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2\,i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2}{1+i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2}{1+c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1+\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\ -c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\ +c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1+\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\ -c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}+$$

$$\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\ +c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1+\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}-\frac{b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+\frac{i\, b^2 \sqrt{x}\, \text{PolyLog}\left[2,\ 1-\frac{(1-i)\left(1+c^{1/4}\sqrt{x}\right)}{1-i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{\sqrt{d\,x}},\ x\right]$$

Problem 92: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{(d\,x)^{3/2}}\,dx$$

Optimal (type 4, 6334 leaves, 197 steps):

$$-\frac{2\sqrt{2}\,a\,b\,c^{1/4}\sqrt{x}\,\text{ArcTan}\left[1-\sqrt{2}\,c^{1/4}\sqrt{x}\right]}{d\sqrt{d\,x}}+\frac{2\sqrt{2}\,a\,b\,c^{1/4}\sqrt{x}\,\text{ArcTan}\left[1+\sqrt{2}\,c^{1/4}\sqrt{x}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,i\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}+\frac{2\,i\,b^2\,c^{1/4}\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}+\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}-\frac{4\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-$$

$$\frac{4\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\left(1-(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+\frac{4\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+$$

$$\frac{4\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-$$

$$\frac{2 b^2 (-c)^{1/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-\frac{2 b^2 (-c)^{1/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1-i) \left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}-$$

$$\frac{4 b^2 c^{1/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}+\frac{2 b^2 (-c)^{1/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right) \left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-$$

$$\frac{2 b^2 (-c)^{1/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-\frac{4 b^2 c^{1/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}+$$

$$\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+$$

$$\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-$$

$$\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}+\frac{4 b^2 c^{1/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}+$$

$$\frac{4 b^2 c^{1/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}-\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-$$

$$\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+$$

$$\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-\frac{2 b^2 c^{1/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+i\,c^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\,\left(1+c^{1/4}\,\sqrt{x}\right)}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{\sqrt{2}\,a\,b\,c^{1/4}\,\sqrt{x}\,\text{Log}\left[1-\sqrt{2}\,c^{1/4}\,\sqrt{x}+\sqrt{c}\,x\right]}{d\,\sqrt{d\,x}}-\frac{\sqrt{2}\,a\,b\,c^{1/4}\,\sqrt{x}\,\text{Log}\left[1+\sqrt{2}\,c^{1/4}\,\sqrt{x}+\sqrt{c}\,x\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{d\,\sqrt{d\,x}}+\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{d\,\sqrt{d\,x}}+\frac{2\,b\,c^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{d\,\sqrt{d\,x}}-\frac{\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{2\,d\,\sqrt{d\,x}}-$$

$$\frac{2\,a\,b\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}+\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}-\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}+\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{b^2\,\text{Log}\left[1-c\,x^2\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}-\frac{b^2\,\text{Log}\left[1+c\,x^2\right]^2}{2\,d\,\sqrt{d\,x}}-\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1-(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,i\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}-\frac{i\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,1+\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{i\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+\frac{i\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{(1+i)\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,i\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1+i\,(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}-\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1+(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{\mathrm{i}\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{(1-\mathrm{i})\left(1+(-c)^{1/4}\sqrt{x}\right)}{1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-\frac{2\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-\mathrm{i}\,c^{1/4}\right)\left(1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,\mathrm{i}\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\mathrm{i}\,c^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-\frac{\mathrm{i}\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\,\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-\mathrm{i}\,c^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\,\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-\mathrm{i}\,c^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-\frac{\mathrm{i}\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}-c^{1/4}\right)\left(1-\mathrm{i}\,c^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}+c^{1/4}\right)\left(1-\mathrm{i}\,c^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+\frac{\mathrm{i}\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{(1+\mathrm{i})\left(1-c^{1/4}\sqrt{x}\right)}{1-\mathrm{i}\,c^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,\mathrm{i}\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1+\mathrm{i}\,c^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-\frac{2\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+\frac{b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{b^2 c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1-\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-\frac{b^2 c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1+\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-$$

$$\frac{b^2 c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1-\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+\frac{b^2 c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1+\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+$$

$$\frac{b^2 c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1-\frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}-\frac{i\, b^2 (-c)^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1-\frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i\, c^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+$$

$$\frac{b^2 (-c)^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1-\frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}+\frac{i\, b^2 c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2, 1-\frac{(1-i) \left(1+c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\, \text{ArcTanh}\left[c x^2\right]\right)^2}{(d x)^{3/2}}, x\right]$$

Problem 93: Unable to integrate problem.

$$\int \frac{\left(a+b\, \text{ArcTanh}\left[c x^2\right]\right)^2}{(d x)^{5/2}} \, dx$$

Optimal (type 4, 6520 leaves, 197 steps):

$$-\frac{2 \sqrt{2}\ a b c^{3/4} \sqrt{x}\ \text{ArcTan}\left[1-\sqrt{2}\ c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}}+\frac{2 \sqrt{2}\ a b c^{3/4} \sqrt{x}\ \text{ArcTan}\left[1+\sqrt{2}\ c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}}-$$

$$\frac{2\, i\, b^2 (-c)^{3/4} \sqrt{x}\ \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}}-\frac{2\, i\, b^2 c^{3/4} \sqrt{x}\ \text{ArcTan}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}}+\frac{2 b^2 (-c)^{3/4} \sqrt{x}\ \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}}+$$

$$\frac{2 b^2 c^{3/4} \sqrt{x}\ \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}}-\frac{4 b^2 (-c)^{3/4} \sqrt{x}\ \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}+$$

$$\frac{4 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}-\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 (-c)^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right) \left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}-$$

$$\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right) \left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}+$$

$$\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1+i) \left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}-\frac{4 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1+i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}+$$

$$\frac{4 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}+\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 (-c)^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}+$$

$$\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}-$$

$$\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 (-c)^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}-$$

$$\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}+\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1-i) \left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}-$$

$$\frac{4 b^2 c^{3/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}-\frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right) \left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}-$$

$$\frac{2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+\frac{4\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}-c^{1/4}\right)\,\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}+c^{1/4}\right)\,\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left(i\,(-c)^{1/4}-c^{1/4}\right)\,\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left(i\,(-c)^{1/4}+c^{1/4}\right)\,\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\,\left(1-c^{1/4}\,\sqrt{x}\right)}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{4\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{4\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+i\,c^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\,\left(1+c^{1/4}\,\sqrt{x}\right)}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{\sqrt{2}\, a\, b\, c^{3/4} \sqrt{x}\, \text{Log}\left[1-\sqrt{2}\, c^{1/4} \sqrt{x}+\sqrt{c}\, x\right]}{3\, d^2 \sqrt{d\, x}}+\frac{\sqrt{2}\, a\, b\, c^{3/4} \sqrt{x}\, \text{Log}\left[1+\sqrt{2}\, c^{1/4} \sqrt{x}+\sqrt{c}\, x\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{2\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1-c\, x^2\right]}{3\, d^2 \sqrt{d\, x}}+\frac{2\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1-c\, x^2\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{2\, b\, c^{3/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \left(2\, a-b\, \text{Log}\left[1-c\, x^2\right]\right)}{3\, d^2 \sqrt{d\, x}}+\frac{2\, b\, c^{3/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \left(2\, a-b\, \text{Log}\left[1-c\, x^2\right]\right)}{3\, d^2 \sqrt{d\, x}}-\frac{\left(2\, a-b\, \text{Log}\left[1-c\, x^2\right]\right)^2}{6\, d^2\, x \sqrt{d\, x}}-$$

$$\frac{2\, a\, b\, \text{Log}\left[1+c\, x^2\right]}{3\, d^2\, x \sqrt{d\, x}}-\frac{2\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1+c\, x^2\right]}{3\, d^2 \sqrt{d\, x}}+\frac{2\, b^2\, c^{3/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1+c\, x^2\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{2\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1+c\, x^2\right]}{3\, d^2 \sqrt{d\, x}}+\frac{2\, b^2\, c^{3/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1+c\, x^2\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{b^2\, \text{Log}\left[1-c\, x^2\right] \text{Log}\left[1+c\, x^2\right]}{3\, d^2\, x \sqrt{d\, x}}-\frac{b^2\, \text{Log}\left[1+c\, x^2\right]^2}{6\, d^2\, x \sqrt{d\, x}}-\frac{2\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{2\, i\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1-i\, (-c)^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}+\frac{i\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1+\frac{2\, (-c)^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{i\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2\, (-c)^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}-\frac{i\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{(1+i) \left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i\, (-c)^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{2\, i\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1+i\, (-c)^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}-\frac{2\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1+\frac{2\, (-c)^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}-\frac{b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2\, (-c)^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1+\frac{2\, (-c)^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+\frac{b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,1-\frac{2\, (-c)^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{\mathrm{i}\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{(1-\mathrm{i})\left(1+(-c)^{1/4}\sqrt{x}\right)}{1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}} - \frac{2\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2}{1-c^{1/4}\sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}} +$$

$$\frac{\mathrm{i}\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-\mathrm{i}\, c^{1/4}\right)\left(1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} + \frac{b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} -$$

$$\frac{2\,\mathrm{i}\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2}{1-\mathrm{i}\, c^{1/4}\sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}} + \frac{\mathrm{i}\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-\mathrm{i}\, c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} +$$

$$\frac{\mathrm{i}\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-\mathrm{i}\, c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} + \frac{\mathrm{i}\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}-c^{1/4}\right)\left(1-\mathrm{i}\, c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} +$$

$$\frac{\mathrm{i}\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}+c^{1/4}\right)\left(1-\mathrm{i}\, c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} - \frac{\mathrm{i}\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{(1+\mathrm{i})\left(1-c^{1/4}\sqrt{x}\right)}{1-\mathrm{i}\, c^{1/4}\sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}} -$$

$$\frac{2\,\mathrm{i}\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2}{1+\mathrm{i}\, c^{1/4}\sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}} - \frac{2\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2}{1+c^{1/4}\sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}} + \frac{b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} +$$

$$\frac{b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} - \frac{b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} -$$

$$\frac{b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} + \frac{b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} +$$

$$\frac{b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} + \frac{\mathrm{i}\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+\mathrm{i}\, c^{1/4}\right)\left(1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}} +$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x} \text{ PolyLog}\left[2, 1-\frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}-\frac{i b^2 c^{3/4} \sqrt{x} \text{ PolyLog}\left[2, 1-\frac{(1-i) \left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}$$

Result (type 8, 22 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{(a+b \text{ ArcTanh}[c x^2])^2}{(d x)^{5/2}}, x\right]$$

Problem 96: Result optimal but 1 more steps used.

$$\int (d x)^m (a+b \text{ ArcTanh}[c x^2]) \, dx$$

Optimal (type 5, 74 leaves, 2 steps) :

$$\frac{(d x)^{1+m} (a+b \text{ ArcTanh}[c x^2])}{d (1+m)}-\frac{2 b c (d x)^{3+m} \text{ Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

Result (type 5, 74 leaves, 3 steps) :

$$\frac{(d x)^{1+m} (a+b \text{ ArcTanh}[c x^2])}{d (1+m)}-\frac{2 b c (d x)^{3+m} \text{ Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^{11} (a+b \text{ ArcTanh}[c x^3])^2 \, dx$$

Optimal (type 3, 125 leaves, 12 steps) :

$$\frac{a b x^3}{6 c^3}+\frac{b^2 x^6}{36 c^2}+\frac{b^2 x^3 \text{ ArcTanh}[c x^3]}{6 c^3}+\frac{b x^9 (a+b \text{ ArcTanh}[c x^3])}{18 c}-\frac{(a+b \text{ ArcTanh}[c x^3])^2}{12 c^4}+\frac{1}{12} x^{12} (a+b \text{ ArcTanh}[c x^3])^2+\frac{b^2 \text{ Log}[1-c^2 x^6]}{9 c^4}$$

Result (type 4, 636 leaves, 62 steps) :

$$\frac{a\,b\,x^3}{12\,c^3}+\frac{23\,b^2\,x^3}{288\,c^3}+\frac{b^2\,x^6}{192\,c^2}-\frac{7\,b^2\,x^9}{864\,c}-\frac{b^2\,x^{12}}{384}+\frac{b^2\,\left(1-c\,x^3\right)^2}{16\,c^4}-\frac{b^2\,\left(1-c\,x^3\right)^3}{54\,c^4}+\frac{b^2\,\left(1-c\,x^3\right)^4}{384\,c^4}-\frac{5\,b^2\,\text{Log}\left[1-c\,x^3\right]}{288\,c^4}+$$
$$\frac{b^2\,\left(1-c\,x^3\right)\,\text{Log}\left[1-c\,x^3\right]}{24\,c^4}+\frac{b^2\,\text{Log}\left[1-c\,x^3\right]^2}{48\,c^4}-\frac{b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{48\,c^2}+\frac{b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{72\,c}-\frac{1}{96}\,b\,x^{12}\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)+$$
$$\frac{1}{48}\,x^{12}\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2-\frac{1}{288}\,b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\left(\frac{48\,\left(1-c\,x^3\right)}{c^4}-\frac{36\,\left(1-c\,x^3\right)^2}{c^4}+\frac{16\,\left(1-c\,x^3\right)^3}{c^4}-\frac{3\,\left(1-c\,x^3\right)^4}{c^4}-\frac{12\,\text{Log}\left[1-c\,x^3\right]}{c^4}\right)-$$
$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{24\,c^4}+\frac{b^2\,\text{Log}\left[1+c\,x^3\right]}{36\,c^4}+\frac{b^2\,x^9\,\text{Log}\left[1+c\,x^3\right]}{36\,c}+\frac{b^2\,\left(1+c\,x^3\right)\,\text{Log}\left[1+c\,x^3\right]}{12\,c^4}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^3\right)\right]\,\text{Log}\left[1+c\,x^3\right]}{24\,c^4}+$$
$$\frac{1}{24}\,b\,x^{12}\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]-\frac{b^2\,\text{Log}\left[1+c\,x^3\right]^2}{48\,c^4}+\frac{1}{48}\,b^2\,x^{12}\,\text{Log}\left[1+c\,x^3\right]^2+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{24\,c^4}+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{24\,c^4}$$

Problem 117: Result valid but suboptimal antiderivative.

$$\int x^8\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2\,dx$$

Optimal (type 4, 146 leaves, 10 steps):

$$\frac{b^2\,x^3}{9\,c^2}-\frac{b^2\,\text{ArcTanh}\left[c\,x^3\right]}{9\,c^3}+\frac{b\,x^6\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)}{9\,c}+\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{9\,c^3}+$$
$$\frac{1}{9}\,x^9\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2-\frac{2\,b\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\,\text{Log}\left[\frac{2}{1-c\,x^3}\right]}{9\,c^3}-\frac{b^2\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x^3}\right]}{9\,c^3}$$

Result (type 4, 536 leaves, 53 steps):

$$-\frac{a\,b\,x^3}{9\,c^2}+\frac{19\,b^2\,x^3}{108\,c^2}-\frac{5\,b^2\,x^6}{216\,c}-\frac{b^2\,x^9}{162}+\frac{b^2\,\left(1-c\,x^3\right)^2}{24\,c^3}-\frac{b^2\,\left(1-c\,x^3\right)^3}{162\,c^3}+\frac{b^2\,\text{Log}\left[1-c\,x^3\right]}{108\,c^3}-\frac{b^2\,\left(1-c\,x^3\right)\,\text{Log}\left[1-c\,x^3\right]}{18\,c^3}+$$
$$\frac{b^2\,\text{Log}\left[1-c\,x^3\right]^2}{36\,c^3}+\frac{b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{36\,c}-\frac{1}{54}\,b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)+\frac{1}{36}\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2-$$
$$\frac{1}{108}\,b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\left(\frac{18\,\left(1-c\,x^3\right)}{c^3}-\frac{9\,\left(1-c\,x^3\right)^2}{c^3}+\frac{2\,\left(1-c\,x^3\right)^3}{c^3}-\frac{6\,\text{Log}\left[1-c\,x^3\right]}{c^3}\right)+\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{18\,c^3}-$$
$$\frac{b^2\,\text{Log}\left[1+c\,x^3\right]}{18\,c^3}+\frac{b^2\,x^6\,\text{Log}\left[1+c\,x^3\right]}{18\,c}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^3\right)\right]\,\text{Log}\left[1+c\,x^3\right]}{18\,c^3}+\frac{1}{18}\,b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]+$$
$$\frac{b^2\,\text{Log}\left[1+c\,x^3\right]^2}{36\,c^3}+\frac{1}{36}\,b^2\,x^9\,\text{Log}\left[1+c\,x^3\right]^2-\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{18\,c^3}+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{18\,c^3}$$

Problem 118: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^5 \left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2 \, dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{a\,b\,x^3}{3\,c}+\frac{b^2\,x^3\,\text{ArcTanh}\left[c\,x^3\right]}{3\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{6\,c^2}+\frac{1}{6}\,x^6\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2+\frac{b^2\,\text{Log}\left[1-c^2\,x^6\right]}{6\,c^2}$$

Result (type 4, 524 leaves, 44 steps):

$$\frac{a\,b\,x^3}{2\,c}-\frac{b^2\,x^6}{24}+\frac{b^2\left(1-c\,x^3\right)^2}{48\,c^2}+\frac{b^2\left(1+c\,x^3\right)^2}{48\,c^2}-\frac{b^2\,\text{Log}\left[1-c\,x^3\right]}{24\,c^2}+\frac{b^2\left(1-c\,x^3\right)\text{Log}\left[1-c\,x^3\right]}{4\,c^2}-\frac{1}{24}\,b\,x^6\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)+$$
$$\frac{b\left(1-c\,x^3\right)^2\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{24\,c^2}-\frac{\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{12\,c^2}+\frac{\left(1-c\,x^3\right)^2\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{24\,c^2}-$$
$$\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^3\right)\right]}{12\,c^2}-\frac{b^2\,\text{Log}\left[1+c\,x^3\right]}{24\,c^2}+\frac{1}{24}\,b^2\,x^6\,\text{Log}\left[1+c\,x^3\right]+\frac{b^2\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]}{4\,c^2}-$$
$$\frac{b^2\left(1+c\,x^3\right)^2\text{Log}\left[1+c\,x^3\right]}{24\,c^2}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^3\right)\right]\text{Log}\left[1+c\,x^3\right]}{12\,c^2}+\frac{1}{12}\,b\,x^6\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]-$$
$$\frac{b^2\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]^2}{12\,c^2}+\frac{b^2\left(1+c\,x^3\right)^2\text{Log}\left[1+c\,x^3\right]^2}{24\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^3\right)\right]}{12\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^3\right)\right]}{12\,c^2}$$

Problem 119: Result valid but suboptimal antiderivative.

$$\int x^2 \left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2 \, dx$$

Optimal (type 4, 96 leaves, 6 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{3\,c}+\frac{1}{3}\,x^3\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2-\frac{2\,b\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\text{Log}\left[\frac{2}{1-c\,x^3}\right]}{3\,c}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x^3}\right]}{3\,c}$$

Result (type 4, 207 leaves, 28 steps):

$$-\frac{\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{12\,c}+\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^3\right)\right]}{6\,c}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^3\right)\right]\text{Log}\left[1+c\,x^3\right]}{6\,c}+$$
$$\frac{1}{6}\,b\,x^3\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]+\frac{b^2\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]^2}{12\,c}-\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^3\right)\right]}{6\,c}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^3\right)\right]}{6\,c}$$

Problem 121: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{x^4}\,dx$$

Optimal (type 4, 90 leaves, 5 steps):

$$\frac{1}{3}c\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{3\,x^3}+\frac{2}{3}b\,c\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\text{Log}\left[2-\frac{2}{1+c\,x^3}\right]-\frac{1}{3}b^2\,c\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x^3}\right]$$

Result (type 4, 237 leaves, 24 steps):

$$2\,a\,b\,c\,\text{Log}[x]-\frac{\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{12\,x^3}-\frac{1}{6}b\,c\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^3\right)\right]-$$
$$\frac{1}{6}b^2\,c\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^3\right)\right]\text{Log}\left[1+c\,x^3\right]-\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]}{6\,x^3}-\frac{b^2\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]^2}{12\,x^3}-$$
$$\frac{1}{3}b^2\,c\,\text{PolyLog}\left[2,-c\,x^3\right]+\frac{1}{3}b^2\,c\,\text{PolyLog}\left[2,c\,x^3\right]+\frac{1}{6}b^2\,c\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^3\right)\right]-\frac{1}{6}b^2\,c\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^3\right)\right]$$

Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{x^7}\,dx$$

Optimal (type 3, 88 leaves, 9 steps):

$$-\frac{b\,c\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)}{3\,x^3}+\frac{1}{6}c^2\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{6\,x^6}+b^2\,c^2\,\text{Log}[x]-\frac{1}{6}b^2\,c^2\,\text{Log}\left[1-c^2\,x^6\right]$$

Result (type 4, 360 leaves, 46 steps):

$$b^2\,c^2\,\text{Log}[x]-\frac{1}{12}b^2\,c^2\,\text{Log}\left[1-c\,x^3\right]-\frac{b\,c\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{12\,x^3}-\frac{b\,c\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{12\,x^3}+$$
$$\frac{1}{24}c^2\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2-\frac{\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{24\,x^6}+\frac{1}{12}b\,c^2\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^3\right)\right]-$$
$$\frac{1}{6}b^2\,c^2\,\text{Log}\left[1+c\,x^3\right]-\frac{b^2\,c\,\text{Log}\left[1+c\,x^3\right]}{6\,x^3}-\frac{1}{12}b^2\,c^2\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^3\right)\right]\text{Log}\left[1+c\,x^3\right]-\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]}{12\,x^6}+$$
$$\frac{1}{24}b^2\,c^2\,\text{Log}\left[1+c\,x^3\right]^2-\frac{b^2\,\text{Log}\left[1+c\,x^3\right]^2}{24\,x^6}-\frac{1}{12}b^2\,c^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^3\right)\right]-\frac{1}{12}b^2\,c^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^3\right)\right]$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x^3])^2}{x^{10}}\,dx$$

Optimal (type 4, 144 leaves, 9 steps):

$$-\frac{b^2 c^2}{9 x^3}+\frac{1}{9} b^2 c^3\,\text{ArcTanh}[c\,x^3]-\frac{b\,c\,(a+b\,\text{ArcTanh}[c\,x^3])}{9 x^6}+\frac{1}{9} c^3\,(a+b\,\text{ArcTanh}[c\,x^3])^2-\frac{(a+b\,\text{ArcTanh}[c\,x^3])^2}{9 x^9}+\frac{2}{9} b\,c^3\,(a+b\,\text{ArcTanh}[c\,x^3])\,\text{Log}\left[2-\frac{2}{1+c\,x^3}\right]-\frac{1}{9} b^2 c^3\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x^3}\right]$$

Result (type 4, 420 leaves, 59 steps):

$$-\frac{b^2 c^2}{9 x^3}+\frac{2}{3} a\,b\,c^3\,\text{Log}[x]-\frac{b\,c\,(2 a-b\,\text{Log}[1-c\,x^3])}{18 x^6}+\frac{b\,c^2\,(2 a-b\,\text{Log}[1-c\,x^3])}{18 x^3}-\frac{b\,c^2\,(1-c\,x^3)\,(2 a-b\,\text{Log}[1-c\,x^3])}{18 x^3}+\frac{1}{36} c^3\,(2 a-b\,\text{Log}[1-c\,x^3])^2-\frac{(2 a-b\,\text{Log}[1-c\,x^3])^2}{36 x^9}-\frac{1}{18} b\,c^3\,(2 a-b\,\text{Log}[1-c\,x^3])\,\text{Log}\left[\frac{1}{2}(1+c\,x^3)\right]+\frac{1}{18} b^2 c^3\,\text{Log}[1+c\,x^3]-\frac{b^2 c\,\text{Log}[1+c\,x^3]}{18 x^6}-\frac{1}{18} b^2 c^3\,\text{Log}\left[\frac{1}{2}(1-c\,x^3)\right]\text{Log}[1+c\,x^3]-\frac{b\,(2 a-b\,\text{Log}[1-c\,x^3])\,\text{Log}[1+c\,x^3]}{18 x^9}-\frac{1}{36} b^2 c^3\,\text{Log}[1+c\,x^3]^2-\frac{b^2\,\text{Log}[1+c\,x^3]^2}{36 x^9}-\frac{1}{9} b^2 c^3\,\text{PolyLog}[2,-c\,x^3]+\frac{1}{9} b^2 c^3\,\text{PolyLog}[2,c\,x^3]+\frac{1}{18} b^2 c^3\,\text{PolyLog}\left[2,\frac{1}{2}(1-c\,x^3)\right]-\frac{1}{18} b^2 c^3\,\text{PolyLog}\left[2,\frac{1}{2}(1+c\,x^3)\right]$$

Problem 124: Result valid but suboptimal antiderivative.

$$\int x^8\,(a+b\,\text{ArcTanh}[c\,x^3])^3\,dx$$

Optimal (type 4, 231 leaves, 13 steps):

$$\frac{a\,b^2 x^3}{3 c^2}+\frac{b^3 x^3\,\text{ArcTanh}[c\,x^3]}{3 c^2}-\frac{b\,(a+b\,\text{ArcTanh}[c\,x^3])^2}{6 c^3}+\frac{b\,x^6\,(a+b\,\text{ArcTanh}[c\,x^3])^2}{6 c}+\frac{(a+b\,\text{ArcTanh}[c\,x^3])^3}{9 c^3}+\frac{1}{9} x^9\,(a+b\,\text{ArcTanh}[c\,x^3])^3-\frac{b\,(a+b\,\text{ArcTanh}[c\,x^3])^2\,\text{Log}\left[\frac{2}{1-c\,x^3}\right]}{3 c^3}+\frac{b^3\,\text{Log}[1-c^2 x^6]}{6 c^3}-\frac{b^2\,(a+b\,\text{ArcTanh}[c\,x^3])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x^3}\right]}{3 c^3}+\frac{b^3\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x^3}\right]}{6 c^3}$$

Result (type 4, 1421 leaves, 239 steps):

$$\frac{2\,a\,b^2\,x^3}{3\,c^2}-\frac{7\,b^3\,x^3}{216\,c^2}-\frac{23\,b^3\,x^6}{432\,c}+\frac{b^3\,x^9}{324}+\frac{b^3\,\left(1-c\,x^3\right)^2}{48\,c^3}+\frac{b^3\,\left(1+c\,x^3\right)^2}{24\,c^3}-\frac{b^3\,\left(1+c\,x^3\right)^3}{324\,c^3}-\frac{b^3\,\text{Log}\left[1-c\,x^3\right]}{24\,c^3}+\frac{b^3\,\left(1-c\,x^3\right)\,\text{Log}\left[1-c\,x^3\right]}{3\,c^3}-$$

$$\frac{b^3\,\text{Log}\left[1-c\,x^3\right]^2}{72\,c^3}-\frac{b^2\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{24\,c}+\frac{b^2\,\left(1-c\,x^3\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{12\,c^3}-\frac{b^2\,\left(1-c\,x^3\right)^3\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{108\,c^3}-$$

$$\frac{1}{72}\,b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2-\frac{b\,\left(1-c\,x^3\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{8\,c^3}+\frac{b\,\left(1-c\,x^3\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{12\,c^3}-\frac{b\,\left(1-c\,x^3\right)^3\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{72\,c^3}-$$

$$\frac{\left(1-c\,x^3\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^3}{24\,c^3}+\frac{\left(1-c\,x^3\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^3}{24\,c^3}-\frac{\left(1-c\,x^3\right)^3\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^3}{72\,c^3}+$$

$$\frac{1}{216}\,b^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\left(\frac{18\,\left(1-c\,x^3\right)}{c^3}-\frac{9\,\left(1-c\,x^3\right)^2}{c^3}+\frac{2\,\left(1-c\,x^3\right)^3}{c^3}-\frac{6\,\text{Log}\left[1-c\,x^3\right]}{c^3}\right)-\frac{b^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{12\,c^3}+$$

$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{12\,c^3}-\frac{7\,b^3\,\text{Log}\left[1+c\,x^3\right]}{108\,c^3}+\frac{b^3\,x^6\,\text{Log}\left[1+c\,x^3\right]}{18\,c}-\frac{1}{108}\,b^3\,x^9\,\text{Log}\left[1+c\,x^3\right]+\frac{11\,b^3\,\left(1+c\,x^3\right)\,\text{Log}\left[1+c\,x^3\right]}{36\,c^3}-$$

$$\frac{b^3\,\left(1+c\,x^3\right)^2\,\text{Log}\left[1+c\,x^3\right]}{12\,c^3}+\frac{b^3\,\left(1+c\,x^3\right)^3\,\text{Log}\left[1+c\,x^3\right]}{108\,c^3}+\frac{b^3\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^3\right)\right]\,\text{Log}\left[1+c\,x^3\right]}{12\,c^3}+\frac{b^2\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]}{12\,c}-$$

$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2\,\text{Log}\left[1+c\,x^3\right]}{24\,c^3}+\frac{1}{24}\,b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2\,\text{Log}\left[1+c\,x^3\right]+\frac{b^3\,\text{Log}\left[1+c\,x^3\right]^2}{72\,c^3}+\frac{1}{72}\,b^3\,x^9\,\text{Log}\left[1+c\,x^3\right]^2-$$

$$\frac{b^3\,\left(1+c\,x^3\right)\,\text{Log}\left[1+c\,x^3\right]^2}{8\,c^3}+\frac{b^3\,\left(1+c\,x^3\right)^2\,\text{Log}\left[1+c\,x^3\right]^2}{12\,c^3}-\frac{b^3\,\left(1+c\,x^3\right)^3\,\text{Log}\left[1+c\,x^3\right]^2}{72\,c^3}+\frac{b^3\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^3\right)\right]\,\text{Log}\left[1+c\,x^3\right]^2}{12\,c^3}+$$

$$\frac{b^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]^2}{24\,c^3}+\frac{1}{24}\,b^2\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]^2+\frac{b^3\,\left(1+c\,x^3\right)\,\text{Log}\left[1+c\,x^3\right]^3}{24\,c^3}-$$

$$\frac{b^3\,\left(1+c\,x^3\right)^2\,\text{Log}\left[1+c\,x^3\right]^3}{24\,c^3}+\frac{b^3\,\left(1+c\,x^3\right)^3\,\text{Log}\left[1+c\,x^3\right]^3}{72\,c^3}+\frac{b^3\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{12\,c^3}-\frac{b^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{6\,c^3}+$$

$$\frac{b^3\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{12\,c^3}+\frac{b^3\,\text{Log}\left[1+c\,x^3\right]\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{6\,c^3}-\frac{b^3\,\text{PolyLog}\left[3,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{6\,c^3}-\frac{b^3\,\text{PolyLog}\left[3,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{6\,c^3}$$

Problem 125: Result valid but suboptimal antiderivative.

$$\int x^5\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^3\,dx$$

Optimal (type 4, 139 leaves, 9 steps):

$$\frac{b\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{2\,c^2}+\frac{b\,x^3\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{2\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^3}{6\,c^2}+\frac{1}{6}x^6\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^3-\frac{b^2\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\text{Log}\left[\frac{2}{1-c\,x^3}\right]}{c^2}-\frac{b^3\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x^3}\right]}{2\,c^2}$$

Result (type 4, 479 leaves, 155 steps):

$$-\frac{b\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{8\,c^2}-\frac{\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^3}{24\,c^2}+\frac{\left(1-c\,x^3\right)^2\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^3}{48\,c^2}+\frac{b^2\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^3\right)\right]}{4\,c^2}+\frac{b^3\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^3\right)\right]\text{Log}\left[1+c\,x^3\right]}{4\,c^2}+\frac{b^2\,x^3\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]}{4\,c}-\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2\text{Log}\left[1+c\,x^3\right]}{16\,c^2}+\frac{1}{16}b\,x^6\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2\text{Log}\left[1+c\,x^3\right]+\frac{b^3\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]^2}{8\,c^2}-\frac{b^2\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]^2}{16\,c^2}+\frac{1}{16}b^2\,x^6\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]^2-\frac{b^3\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]^3}{24\,c^2}+\frac{b^3\left(1+c\,x^3\right)^2\text{Log}\left[1+c\,x^3\right]^3}{48\,c^2}-\frac{b^3\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^3\right)\right]}{4\,c^2}+\frac{b^3\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^3\right)\right]}{4\,c^2}$$

Problem 126: Result valid but suboptimal antiderivative.

$$\int x^2\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^3\,dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^3}{3\,c}+\frac{1}{3}x^3\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^3-\frac{b\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2\text{Log}\left[\frac{2}{1-c\,x^3}\right]}{c}-\frac{b^2\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\text{PolyLog}\left[2,1-\frac{2}{1-c\,x^3}\right]}{c}+\frac{b^3\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x^3}\right]}{2\,c}$$

Result (type 4, 390 leaves, 82 steps):

$$-\frac{\left(1-c x^3\right)\left(2 a-b \text{Log}\left[1-c x^3\right]\right)^3}{24 c}+\frac{b\left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2 \text{Log}\left[\frac{1}{2}\left(1+c x^3\right)\right]}{4 c}-\frac{b\left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2 \text{Log}\left[1+c x^3\right]}{8 c}+$$
$$\frac{1}{8} b x^3\left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2 \text{Log}\left[1+c x^3\right]+\frac{b^3 \text{Log}\left[\frac{1}{2}\left(1-c x^3\right)\right] \text{Log}\left[1+c x^3\right]^2}{4 c}+\frac{b^2\left(2 a-b \text{Log}\left[1-c x^3\right]\right) \text{Log}\left[1+c x^3\right]^2}{8 c}+$$
$$\frac{1}{8} b^2 x^3\left(2 a-b \text{Log}\left[1-c x^3\right]\right) \text{Log}\left[1+c x^3\right]^2+\frac{b^3\left(1+c x^3\right) \text{Log}\left[1+c x^3\right]^3}{24 c}-\frac{b^2\left(2 a-b \text{Log}\left[1-c x^3\right]\right) \text{PolyLog}\left[2, \frac{1}{2}\left(1-c x^3\right)\right]}{2 c}+$$
$$\frac{b^3 \text{Log}\left[1+c x^3\right] \text{PolyLog}\left[2, \frac{1}{2}\left(1+c x^3\right)\right]}{2 c}-\frac{b^3 \text{PolyLog}\left[3, \frac{1}{2}\left(1-c x^3\right)\right]}{2 c}-\frac{b^3 \text{PolyLog}\left[3, \frac{1}{2}\left(1+c x^3\right)\right]}{2 c}$$

Problem 128: Unable to integrate problem.

$$\int \frac{\left(a+b \text{ArcTanh}\left[c x^3\right]\right)^3}{x^4} \, dx$$

Optimal (type 4, 120 leaves, 6 steps):

$$\frac{1}{3} c\left(a+b \text{ArcTanh}\left[c x^3\right]\right)^3-\frac{\left(a+b \text{ArcTanh}\left[c x^3\right]\right)^3}{3 x^3}+b c\left(a+b \text{ArcTanh}\left[c x^3\right]\right)^2 \text{Log}\left[2-\frac{2}{1+c x^3}\right]-$$
$$b^2 c\left(a+b \text{ArcTanh}\left[c x^3\right]\right) \text{PolyLog}\left[2,-1+\frac{2}{1+c x^3}\right]-\frac{1}{2} b^3 c \text{PolyLog}\left[3,-1+\frac{2}{1+c x^3}\right]$$

Result (type 8, 284 leaves, 16 steps):

$$\frac{1}{8} b c \text{Log}\left[c x^3\right]\left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2-\frac{\left(1-c x^3\right)\left(2 a-b \text{Log}\left[1-c x^3\right]\right)^3}{24 x^3}+$$
$$\frac{1}{8} b^3 c \text{Log}\left[-c x^3\right] \text{Log}\left[1+c x^3\right]^2-\frac{b^3\left(1+c x^3\right) \text{Log}\left[1+c x^3\right]^3}{24 x^3}-\frac{1}{4} b^2 c\left(2 a-b \text{Log}\left[1-c x^3\right]\right) \text{PolyLog}\left[2, 1-c x^3\right]+$$
$$\frac{1}{4} b^3 c \text{Log}\left[1+c x^3\right] \text{PolyLog}\left[2, 1+c x^3\right]-\frac{1}{4} b^3 c \text{PolyLog}\left[3, 1-c x^3\right]-\frac{1}{4} b^3 c \text{PolyLog}\left[3, 1+c x^3\right]+$$
$$\frac{3}{8} b \text{Unintegrable}\left[\frac{\left(-2 a+b \text{Log}\left[1-c x^3\right]\right)^2 \text{Log}\left[1+c x^3\right]}{x^4}, x\right]-\frac{3}{8} b^2 \text{Unintegrable}\left[\frac{\left(-2 a+b \text{Log}\left[1-c x^3\right]\right) \text{Log}\left[1+c x^3\right]^2}{x^4}, x\right]$$

Problem 129: Unable to integrate problem.

$$\int \frac{\left(a+b \text{ArcTanh}\left[c x^3\right]\right)^3}{x^7} \, dx$$

Optimal (type 4, 136 leaves, 8 steps):

$$\frac{1}{2} b c^2 \left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^2-\frac{b c \left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^2}{2 x^3}+\frac{1}{6} c^2 \left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^3-\frac{\left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^3}{6 x^6}+b^2 c^2 \left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right) \operatorname{Log}\left[2-\frac{2}{1+c x^3}\right]-\frac{1}{2} b^3 c^2 \operatorname{PolyLog}\left[2,-1+\frac{2}{1+c x^3}\right]$$

Result (type 8, 437 leaves, 29 steps) :

$$\frac{3}{4} a b^2 c^2 \operatorname{Log}[x]-\frac{b c \left(1-c x^3\right) \left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2}{16 x^3}+\frac{1}{16} b c^2 \operatorname{Log}\left[c x^3\right] \left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2+\frac{1}{48} c^2 \left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^3-\frac{\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^3}{48 x^6}-\frac{b^3 c \left(1+c x^3\right) \operatorname{Log}\left[1+c x^3\right]^2}{16 x^3}-\frac{1}{16} b^3 c^2 \operatorname{Log}\left[-c x^3\right] \operatorname{Log}\left[1+c x^3\right]^2+\frac{1}{48} b^3 c^2 \operatorname{Log}\left[1+c x^3\right]^3-\frac{b^3 \operatorname{Log}\left[1+c x^3\right]^3}{48 x^6}-\frac{1}{8} b^3 c^2 \operatorname{PolyLog}\left[2,-c x^3\right]+\frac{1}{8} b^3 c^2 \operatorname{PolyLog}\left[2,c x^3\right]-\frac{1}{8} b^2 c^2 \left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{PolyLog}\left[2,1-c x^3\right]-\frac{1}{8} b^3 c^2 \operatorname{Log}\left[1+c x^3\right] \operatorname{PolyLog}\left[2,1+c x^3\right]-\frac{1}{8} b^3 c^2 \operatorname{PolyLog}\left[3,1-c x^3\right]+\frac{1}{8} b^3 c^2 \operatorname{PolyLog}\left[3,1+c x^3\right]+\frac{3}{8} b \operatorname{Unintegrable}\left[\frac{\left(-2 a+b \operatorname{Log}\left[1-c x^3\right]\right)^2 \operatorname{Log}\left[1+c x^3\right]}{x^7},x\right]-\frac{3}{8} b^2 \operatorname{Unintegrable}\left[\frac{\left(-2 a+b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{Log}\left[1+c x^3\right]^2}{x^7},x\right]$$

Problem 132: Result optimal but 1 more steps used.

$$\int (d x)^m \left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right) dx$$

Optimal (type 5, 74 leaves, 2 steps) :

$$\frac{(d x)^{1+m} \left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)}{d (1+m)}-\frac{3 b c (d x)^{4+m} \operatorname{Hypergeometric2F1}\left[1,\frac{4+m}{6},\frac{10+m}{6},c^2 x^6\right]}{d^4 (1+m) (4+m)}$$

Result (type 5, 74 leaves, 3 steps) :

$$\frac{(d x)^{1+m} \left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)}{d (1+m)}-\frac{3 b c (d x)^{4+m} \operatorname{Hypergeometric2F1}\left[1,\frac{4+m}{6},\frac{10+m}{6},c^2 x^6\right]}{d^4 (1+m) (4+m)}$$

Problem 143: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \left(a+b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 3, 123 leaves, 14 steps) :

$$\frac{1}{12} b^2 c^2 x^2 + \frac{1}{2} b c^3 x \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) + \frac{1}{6} b c x^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) - \frac{1}{4} c^4 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{4} x^4 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{3} b^2 c^4 \operatorname{Log}\left[1 - \frac{c^2}{x^2}\right] + \frac{2}{3} b^2 c^4 \operatorname{Log}[x]$$

Result (type 4, 812 leaves, 88 steps):

$$\frac{1}{4} a b c^3 x - \frac{1}{8} a b c^2 x^2 + \frac{1}{12} b^2 c^2 x^2 + \frac{1}{12} a b c x^3 + \frac{5}{48} b^2 c^4 \operatorname{Log}\left[1 - \frac{c}{x}\right] - \frac{1}{8} b^2 c^3 x \operatorname{Log}\left[1 - \frac{c}{x}\right] + \frac{1}{16} b^2 c^2 x^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] - \frac{1}{24} b^2 c x^3 \operatorname{Log}\left[1 - \frac{c}{x}\right] + \frac{1}{8} b c^3 \left(1 - \frac{c}{x}\right) x \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) + \frac{1}{16} b c^2 x^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) + \frac{1}{24} b c x^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) - \frac{1}{16} c^4 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{16} x^4 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{8} b^2 c^3 x \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{16} b^2 c^2 x^2 \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{24} b^2 c x^3 \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{4} a b x^4 \operatorname{Log}\left[1 + \frac{c}{x}\right] - \frac{1}{8} b^2 x^4 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{5}{48} b^2 c^4 \operatorname{Log}[c - x] + \frac{1}{8} b^2 c^4 \operatorname{Log}\left[1 + \frac{c}{x}\right] \operatorname{Log}[c - x] + \frac{1}{4} a b c^4 \operatorname{Log}[x] + \frac{11}{24} b^2 c^4 \operatorname{Log}[x] + \frac{1}{8} b^2 c^4 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{x}{c}\right] - \frac{1}{4} a b c^4 \operatorname{Log}[c + x] + \frac{5}{48} b^2 c^4 \operatorname{Log}[c + x] + \frac{1}{8} b^2 c^4 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}[c + x] - \frac{1}{8} b^2 c^4 \operatorname{Log}\left[\frac{c - x}{2 c}\right] \operatorname{Log}[c + x] + \frac{1}{8} b^2 c^4 \operatorname{Log}\left[-\frac{x}{c}\right] \operatorname{Log}[c + x] - \frac{1}{8} b^2 c^4 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{c + x}{2 c}\right] + \frac{11}{48} b^2 c^4 \operatorname{Log}\left[\frac{c + x}{x}\right] + \frac{1}{8} b^2 c^3 x \operatorname{Log}\left[\frac{c + x}{x}\right] - \frac{1}{16} b^2 c^2 x^2 \operatorname{Log}\left[\frac{c + x}{x}\right] + \frac{1}{24} b^2 c x^3 \operatorname{Log}\left[\frac{c + x}{x}\right] - \frac{1}{16} b^2 c^4 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{16} b^2 x^4 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, \frac{c - x}{2 c}\right] - \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, \frac{c}{x}\right] - \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, \frac{c + x}{2 c}\right] + \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, 1 - \frac{x}{c}\right] + \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, 1 + \frac{x}{c}\right]$$

Problem 144: Result valid but suboptimal antiderivative.

$$\int x^2 \left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{1}{3} b^2 c^2 x - \frac{1}{3} b^2 c^3 \operatorname{ArcCoth}\left[\frac{x}{c}\right] + \frac{1}{3} b c x^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) - \frac{1}{3} c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{3} x^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 - \frac{2}{3} b c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{Log}\left[2 - \frac{2}{1 + \frac{c}{x}}\right] + \frac{1}{3} b^2 c^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + \frac{c}{x}}\right]$$

Result (type 4, 695 leaves, 73 steps):

$$-\frac{1}{3} a b c^2 x + \frac{1}{3} b^2 c^2 x + \frac{1}{6} a b c x^2 + \frac{1}{12} b^2 c^3 \text{Log}\left[1-\frac{c}{x}\right] + \frac{1}{6} b^2 c^2 x \text{Log}\left[1-\frac{c}{x}\right] - \frac{1}{12} b^2 c x^2 \text{Log}\left[1-\frac{c}{x}\right] +$$
$$\frac{1}{6} b c^2 \left(1-\frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right) + \frac{1}{12} b c x^2 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right) - \frac{1}{12} c^3 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 + \frac{1}{12} x^3 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 +$$
$$\frac{1}{6} b^2 c^2 x \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{12} b^2 c x^2 \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{3} a b x^3 \text{Log}\left[1+\frac{c}{x}\right] - \frac{1}{6} b^2 x^3 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right] - \frac{1}{12} b^2 c^3 \text{Log}[c-x] +$$
$$\frac{1}{6} b^2 c^3 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x] + \frac{1}{3} a b c^3 \text{Log}[x] + \frac{1}{6} b^2 c^3 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right] + \frac{1}{3} a b c^3 \text{Log}[c+x] + \frac{1}{12} b^2 c^3 \text{Log}[c+x] -$$
$$\frac{1}{6} b^2 c^3 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x] + \frac{1}{6} b^2 c^3 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x] - \frac{1}{6} b^2 c^3 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x] - \frac{1}{6} b^2 c^3 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right] -$$
$$\frac{1}{4} b^2 c^3 \text{Log}\left[\frac{c+x}{x}\right] - \frac{1}{6} b^2 c^2 x \text{Log}\left[\frac{c+x}{x}\right] + \frac{1}{12} b^2 c x^2 \text{Log}\left[\frac{c+x}{x}\right] + \frac{1}{12} b^2 c^3 \text{Log}\left[\frac{c+x}{x}\right]^2 + \frac{1}{12} b^2 x^3 \text{Log}\left[\frac{c+x}{x}\right]^2 - \frac{1}{6} b^2 c^3 \text{PolyLog}\left[2, \frac{c-x}{2 c}\right] +$$
$$\frac{1}{6} b^2 c^3 \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{1}{6} b^2 c^3 \text{PolyLog}\left[2, \frac{c}{x}\right] + \frac{1}{6} b^2 c^3 \text{PolyLog}\left[2, \frac{c+x}{2 c}\right] + \frac{1}{6} b^2 c^3 \text{PolyLog}\left[2, 1-\frac{x}{c}\right] - \frac{1}{6} b^2 c^3 \text{PolyLog}\left[2, 1+\frac{x}{c}\right]$$

Problem 145: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 3, 83 leaves, 9 steps):

$$b c x \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right) - \frac{1}{2} c^2 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{2} x^2 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{2} b^2 c^2 \text{Log}\left[1-\frac{c^2}{x^2}\right] + b^2 c^2 \text{Log}[x]$$

Result (type 4, 574 leaves, 58 steps):

$$\frac{1}{2} a b c x - \frac{1}{4} b^2 c x \text{Log}\left[1-\frac{c}{x}\right] + \frac{1}{4} b c \left(1-\frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right) - \frac{1}{8} c^2 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 + \frac{1}{8} x^2 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 +$$
$$\frac{1}{4} b^2 c x \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{2} a b x^2 \text{Log}\left[1+\frac{c}{x}\right] - \frac{1}{4} b^2 x^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{4} b^2 c^2 \text{Log}[c-x] + \frac{1}{4} b^2 c^2 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x] +$$
$$\frac{1}{2} a b c^2 \text{Log}[x] + \frac{1}{2} b^2 c^2 \text{Log}[x] + \frac{1}{4} b^2 c^2 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right] - \frac{1}{2} a b c^2 \text{Log}[c+x] + \frac{1}{4} b^2 c^2 \text{Log}[c+x] + \frac{1}{4} b^2 c^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x] -$$
$$\frac{1}{4} b^2 c^2 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x] + \frac{1}{4} b^2 c^2 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x] - \frac{1}{4} b^2 c^2 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right] + \frac{1}{4} b^2 c^2 \text{Log}\left[\frac{c+x}{x}\right] +$$
$$\frac{1}{4} b^2 c x \text{Log}\left[\frac{c+x}{x}\right] - \frac{1}{8} b^2 c^2 \text{Log}\left[\frac{c+x}{x}\right]^2 + \frac{1}{8} b^2 x^2 \text{Log}\left[\frac{c+x}{x}\right]^2 - \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, \frac{c-x}{2 c}\right] - \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, -\frac{c}{x}\right] -$$
$$\frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, \frac{c}{x}\right] - \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, \frac{c+x}{2 c}\right] + \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, 1-\frac{x}{c}\right] + \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, 1+\frac{x}{c}\right]$$

Problem 146: Result valid but suboptimal antiderivative.

$$\int \left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$c\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+x\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2-2\,b\,c\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\,\text{Log}\left[\frac{2\,c}{c-x}\right]-b^2\,c\,\text{PolyLog}\left[2,\,-\frac{c+x}{c-x}\right]$$

Result (type 4, 370 leaves, 31 steps):

$$a^2\,x-a\,b\,x\,\text{Log}\left[1-\frac{c}{x}\right]-\frac{1}{4}\,b^2\,(c-x)\,\text{Log}\left[1-\frac{c}{x}\right]^2+a\,b\,x\,\text{Log}\left[1+\frac{c}{x}\right]-\frac{1}{2}\,b^2\,x\,\text{Log}\left[1-\frac{c}{x}\right]\,\text{Log}\left[1+\frac{c}{x}\right]+\frac{1}{4}\,b^2\,(c+x)\,\text{Log}\left[1+\frac{c}{x}\right]^2-$$
$$\frac{1}{2}\,b^2\,c\,\text{Log}\left[1-\frac{c}{x}\right]\,\text{Log}[-c-x]+a\,b\,c\,\text{Log}[c-x]+\frac{1}{2}\,b^2\,c\,\text{Log}[-c-x]\,\text{Log}\left[\frac{c-x}{2\,c}\right]-\frac{1}{2}\,b^2\,c\,\text{Log}[-c-x]\,\text{Log}\left[-\frac{x}{c}\right]+$$
$$\frac{1}{2}\,b^2\,c\,\text{Log}\left[1+\frac{c}{x}\right]\,\text{Log}[-c+x]+\frac{1}{2}\,b^2\,c\,\text{Log}\left[\frac{x}{c}\right]\,\text{Log}[-c+x]+a\,b\,c\,\text{Log}[c+x]-\frac{1}{2}\,b^2\,c\,\text{Log}[-c+x]\,\text{Log}\left[\frac{c+x}{2\,c}\right]-\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,\frac{c-x}{2\,c}\right]+$$
$$\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,-\frac{c}{x}\right]-\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,\frac{c}{x}\right]+\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,\frac{c+x}{2\,c}\right]+\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,1-\frac{x}{c}\right]-\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,1+\frac{x}{c}\right]$$

Problem 148: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^2}{x^2}\,dx$$

Optimal (type 4, 87 leaves, 6 steps):

$$-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{c}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{x}+\frac{2\,b\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\,\text{Log}\left[\frac{2}{1-\frac{c}{x}}\right]}{c}+\frac{b^2\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\frac{c}{x}}\right]}{c}$$

Result (type 4, 205 leaves, 28 steps):

$$\frac{\left(1-\frac{c}{x}\right)\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{4\,c}-\frac{b\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\,\text{Log}\left[\frac{c+x}{2\,x}\right]}{2\,c}-\frac{b\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,x}-$$
$$\frac{b^2\,\text{Log}\left[-\frac{c-x}{2\,x}\right]\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,c}-\frac{b^2\,\left(1+\frac{c}{x}\right)\,\text{Log}\left[\frac{c+x}{x}\right]^2}{4\,c}+\frac{b^2\,\text{PolyLog}\left[2,\,-\frac{c-x}{2\,x}\right]}{2\,c}-\frac{b^2\,\text{PolyLog}\left[2,\,\frac{c+x}{2\,x}\right]}{2\,c}$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^2}{x^3}\,dx$$

Optimal (type 3, 87 leaves, 7 steps):

$$-\frac{a\,b}{c\,x}-\frac{b^2\,\text{ArcCoth}\left[\frac{x}{c}\right]}{c\,x}+\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2\,c^2}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2\,x^2}-\frac{b^2\,\text{Log}\left[1-\frac{c^2}{x^2}\right]}{2\,c^2}$$

Result (type 4, 707 leaves, 66 steps):

$$-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16\,c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16\,c^2}+\frac{a\,b}{4\,x^2}+\frac{b^2}{8\,x^2}-\frac{3\,a\,b}{2\,c\,x}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]}{8\,c^2}-\frac{3\,b^2\left(1-\frac{c}{x}\right)\text{Log}\left[1-\frac{c}{x}\right]}{4\,c^2}-\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]}{8\,x^2}-\frac{b\left(1-\frac{c}{x}\right)^2\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)}{8\,c^2}+$$
$$\frac{\left(1-\frac{c}{x}\right)\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{4\,c^2}-\frac{\left(1-\frac{c}{x}\right)^2\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{8\,c^2}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}\left[1+\frac{c}{x}\right]}{4\,x^2}-\frac{b^2\,\text{Log}\left[1+\frac{c}{x}\right]\text{Log}[c-x]}{4\,c^2}-\frac{b^2\,\text{Log}[c-x]\,\text{Log}\left[\frac{x}{c}\right]}{4\,c^2}-$$
$$\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}[c+x]}{4\,c^2}+\frac{b^2\,\text{Log}\left[\frac{c-x}{2\,c}\right]\text{Log}[c+x]}{4\,c^2}-\frac{b^2\,\text{Log}\left[-\frac{x}{c}\right]\text{Log}[c+x]}{4\,c^2}+\frac{b^2\,\text{Log}[c-x]\,\text{Log}\left[\frac{c+x}{2\,c}\right]}{4\,c^2}+\frac{a\,b\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,c^2}+\frac{b^2\,\text{Log}\left[\frac{c+x}{x}\right]}{8\,c^2}-$$
$$\frac{3\,b^2\left(1+\frac{c}{x}\right)\text{Log}\left[\frac{c+x}{x}\right]}{4\,c^2}+\frac{b^2\left(1+\frac{c}{x}\right)^2\text{Log}\left[\frac{c+x}{x}\right]}{8\,c^2}-\frac{a\,b\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,x^2}-\frac{b^2\,\text{Log}\left[\frac{c+x}{x}\right]}{8\,x^2}+\frac{b^2\left(1+\frac{c}{x}\right)\text{Log}\left[\frac{c+x}{x}\right]^2}{4\,c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2\text{Log}\left[\frac{c+x}{x}\right]^2}{8\,c^2}+$$
$$\frac{b^2\,\text{PolyLog}\left[2,\frac{c-x}{2\,c}\right]}{4\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,-\frac{c}{x}\right]}{4\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c}{x}\right]}{4\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c+x}{2\,c}\right]}{4\,c^2}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{x}{c}\right]}{4\,c^2}-\frac{b^2\,\text{PolyLog}\left[2,1+\frac{x}{c}\right]}{4\,c^2}$$

Problem 150: Unable to integrate problem.

$$\int x^3\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3\,dx$$

Optimal (type 4, 203 leaves, 17 steps):

$$\frac{1}{4}\,b^3\,c^3\,x-\frac{1}{4}\,b^3\,c^4\,\text{ArcCoth}\left[\frac{x}{c}\right]+\frac{1}{4}\,b^2\,c^2\,x^2\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)-b\,c^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+$$
$$\frac{3}{4}\,b\,c^3\,x\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+\frac{1}{4}\,b\,c\,x^3\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2-\frac{1}{4}\,c^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3+$$
$$\frac{1}{4}\,x^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3-2\,b^2\,c^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\text{Log}\left[2-\frac{2}{1+\frac{c}{x}}\right]+b^3\,c^4\,\text{PolyLog}\left[2,-1+\frac{2}{1+\frac{c}{x}}\right]$$

Result (type 8, 1398 leaves, 139 steps) :

$$\frac{3}{8} a^2 b c^3 x - \frac{5}{16} a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{3}{16} a b^2 c^2 x^2 + \frac{1}{8} a^2 b c x^3 + \frac{3}{8} b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1-\frac{c}{x}\right]^2 \text{Log}\left[1+\frac{c}{x}\right], x\right] -$$
$$\frac{3}{8} b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]^2, x\right] + \frac{1}{32} b^3 c^4 \text{Log}\left[1-\frac{c}{x}\right] - \frac{3}{8} a b^2 c^3 x \text{Log}\left[1-\frac{c}{x}\right] + \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[1-\frac{c}{x}\right] -$$
$$\frac{1}{8} a b^2 c x^3 \text{Log}\left[1-\frac{c}{x}\right] + \frac{5}{32} b^2 c^3 \left(1-\frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right) + \frac{1}{32} b^2 c^2 x^2 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right) - \frac{5}{64} b c^4 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 +$$
$$\frac{3}{32} b c^3 \left(1-\frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 + \frac{3}{64} b c^2 x^2 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 + \frac{1}{32} b c x^3 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 - \frac{1}{32} c^4 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^3 +$$
$$\frac{1}{32} x^4 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^3 + \frac{3}{8} a b^2 c^3 x \text{Log}\left[1+\frac{c}{x}\right] + \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{8} a b^2 c x^3 \text{Log}\left[1+\frac{c}{x}\right] + \frac{3}{8} a^2 b x^4 \text{Log}\left[1+\frac{c}{x}\right] -$$
$$\frac{3}{8} a b^2 x^4 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right] + \frac{5}{16} a b^2 c^4 \text{Log}[c-x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x] - \frac{3}{32} b c^4 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c}{x}\right] +$$
$$\frac{11}{8} a b^2 c^4 \text{Log}[x] + \frac{3}{8} a b^2 c^4 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right] - \frac{3}{8} a^2 b c^4 \text{Log}[c+x] + \frac{5}{16} a b^2 c^4 \text{Log}[c+x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x] -$$
$$\frac{3}{8} a b^2 c^4 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x] - \frac{3}{8} a b^2 c^4 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right] + \frac{11}{16} a b^2 c^4 \text{Log}\left[\frac{c+x}{x}\right] -$$
$$\frac{1}{32} b^3 c^4 \text{Log}\left[\frac{c+x}{x}\right] + \frac{3}{8} a b^2 c^3 x \text{Log}\left[\frac{c+x}{x}\right] - \frac{5}{32} b^3 c^3 \left(1+\frac{c}{x}\right) x \text{Log}\left[\frac{c+x}{x}\right] - \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[\frac{c+x}{x}\right] + \frac{1}{32} b^3 c^2 x^2 \text{Log}\left[\frac{c+x}{x}\right] +$$
$$\frac{1}{8} a b^2 c x^3 \text{Log}\left[\frac{c+x}{x}\right] - \frac{3}{16} a b^2 c^4 \text{Log}\left[\frac{c+x}{x}\right]^2 + \frac{5}{64} b^3 c^4 \text{Log}\left[\frac{c+x}{x}\right]^2 + \frac{3}{32} b^3 c^3 \left(1+\frac{c}{x}\right) x \text{Log}\left[\frac{c+x}{x}\right]^2 - \frac{3}{64} b^3 c^2 x^2 \text{Log}\left[\frac{c+x}{x}\right]^2 +$$
$$\frac{1}{32} b^3 c x^3 \text{Log}\left[\frac{c+x}{x}\right]^2 + \frac{3}{16} a b^2 x^4 \text{Log}\left[\frac{c+x}{x}\right]^2 + \frac{3}{32} b^3 c^4 \text{Log}\left[-\frac{c}{x}\right] \text{Log}\left[\frac{c+x}{x}\right]^2 - \frac{1}{32} b^3 c^4 \text{Log}\left[\frac{c+x}{x}\right]^3 + \frac{1}{32} b^3 x^4 \text{Log}\left[\frac{c+x}{x}\right]^3 +$$
$$\frac{3}{16} b^2 c^4 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right) \text{PolyLog}\left[2, 1-\frac{c}{x}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c-x}{2 c}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, -\frac{c}{x}\right] +$$
$$\frac{11}{32} b^3 c^4 \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{11}{32} b^3 c^4 \text{PolyLog}\left[2, \frac{c}{x}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c+x}{2 c}\right] + \frac{3}{16} b^3 c^4 \text{Log}\left[\frac{c+x}{x}\right] \text{PolyLog}\left[2, \frac{c+x}{x}\right] +$$
$$\frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1-\frac{x}{c}\right] + \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1+\frac{x}{c}\right] + \frac{3}{16} b^3 c^4 \text{PolyLog}\left[3, 1-\frac{c}{x}\right] - \frac{3}{16} b^3 c^4 \text{PolyLog}\left[3, \frac{c+x}{x}\right]$$

Problem 151: Unable to integrate problem.

$$\int x^2 \left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 217 leaves, 15 steps) :

$$b^2 c^2 x \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)-\frac{1}{2} b c^3 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+\frac{1}{2} b c x^2 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2-\frac{1}{3} c^3 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3+\frac{1}{3} x^3 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3-b c^3 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \text{Log}\left[2-\frac{2}{1+\frac{c}{x}}\right]+\frac{1}{2} b^3 c^3 \text{Log}\left[1-\frac{c^2}{x^2}\right]+b^3 c^3 \text{Log}[x]+b^2 c^3 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{PolyLog}\left[2,-1+\frac{2}{1+\frac{c}{x}}\right]+\frac{1}{2} b^3 c^3 \text{PolyLog}\left[3,-1+\frac{2}{1+\frac{c}{x}}\right]$$

Result (type 8, 1152 leaves, 103 steps):

$$-\frac{1}{2} a^2 b c^2 x+\frac{3}{4} a b^2 c^2 x+\frac{1}{4} a^2 b c x^2+\frac{3}{8} b^3 \text{CannotIntegrate}\left[x^2 \text{Log}\left[1-\frac{c}{x}\right]^2 \text{Log}\left[1+\frac{c}{x}\right],x\right]-\frac{3}{8} b^3 \text{CannotIntegrate}\left[x^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]^2,x\right]+\frac{1}{2} a b^2 c^2 x \text{Log}\left[1-\frac{c}{x}\right]-\frac{1}{4} a b^2 c x^2 \text{Log}\left[1-\frac{c}{x}\right]+\frac{1}{8} b^2 c^2 \left(1-\frac{c}{x}\right) x \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)-\frac{1}{16} b c^3 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2+\frac{1}{8} b c^2 \left(1-\frac{c}{x}\right) x \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2+\frac{1}{16} b c x^2 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2-\frac{1}{24} c^3 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^3+\frac{1}{24} x^3 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^3+\frac{1}{2} a b^2 c^2 x \text{Log}\left[1+\frac{c}{x}\right]+\frac{1}{4} a b^2 c x^2 \text{Log}\left[1+\frac{c}{x}\right]+\frac{1}{2} a^2 b x^3 \text{Log}\left[1+\frac{c}{x}\right]-\frac{1}{2} a b^2 x^3 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]-\frac{1}{4} a b^2 c^3 \text{Log}[c-x]+\frac{1}{2} a b^2 c^3 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x]-\frac{1}{8} b c^3 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c}{x}\right]+\frac{1}{4} b^3 c^3 \text{Log}[x]+\frac{1}{2} a b^2 c^3 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right]+\frac{1}{2} a^2 b c^3 \text{Log}[c+x]+\frac{1}{4} a b^2 c^3 \text{Log}[c+x]-\frac{1}{2} a b^2 c^3 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x]+\frac{1}{2} a b^2 c^3 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x]-\frac{1}{2} a b^2 c^3 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x]-\frac{1}{2} a b^2 c^3 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right]-\frac{3}{4} a b^2 c^3 \text{Log}\left[\frac{c+x}{x}\right]-\frac{1}{2} a b^2 c^2 x \text{Log}\left[\frac{c+x}{x}\right]+\frac{1}{8} b^3 c^2 \left(1+\frac{c}{x}\right) x \text{Log}\left[\frac{c+x}{x}\right]+\frac{1}{4} a b^2 c x^2 \text{Log}\left[\frac{c+x}{x}\right]+\frac{1}{4} a b^2 c^3 \text{Log}\left[\frac{c+x}{x}\right]^2-\frac{1}{16} b^3 c^3 \text{Log}\left[\frac{c+x}{x}\right]^2-\frac{1}{8} b^3 c^2 \left(1+\frac{c}{x}\right) x \text{Log}\left[\frac{c+x}{x}\right]^2+\frac{1}{16} b^3 c x^2 \text{Log}\left[\frac{c+x}{x}\right]^2+\frac{1}{4} a b^2 x^3 \text{Log}\left[\frac{c+x}{x}\right]^2-\frac{1}{8} b^3 c^3 \text{Log}\left[-\frac{c}{x}\right] \text{Log}\left[\frac{c+x}{x}\right]^2+\frac{1}{24} b^3 c^3 \text{Log}\left[\frac{c+x}{x}\right]^3+\frac{1}{24} b^3 x^3 \text{Log}\left[\frac{c+x}{x}\right]^3+\frac{1}{4} b^2 c^3 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right) \text{PolyLog}\left[2,1-\frac{c}{x}\right]-\frac{1}{2} a b^2 c^3 \text{PolyLog}\left[2,\frac{c-x}{2 c}\right]+\frac{1}{2} a b^2 c^3 \text{PolyLog}\left[2,-\frac{c}{x}\right]-\frac{3}{8} b^3 c^3 \text{PolyLog}\left[2,-\frac{c}{x}\right]-\frac{3}{8} b^3 c^3 \text{PolyLog}\left[2,\frac{c}{x}\right]+\frac{1}{2} a b^2 c^3 \text{PolyLog}\left[2,\frac{c+x}{2 c}\right]-\frac{1}{4} b^3 c^3 \text{Log}\left[\frac{c+x}{x}\right] \text{PolyLog}\left[2,\frac{c+x}{x}\right]+\frac{1}{2} a b^2 c^3 \text{PolyLog}\left[2,1-\frac{x}{c}\right]-\frac{1}{2} a b^2 c^3 \text{PolyLog}\left[2,1+\frac{x}{c}\right]+\frac{1}{4} b^3 c^3 \text{PolyLog}\left[3,1-\frac{c}{x}\right]+\frac{1}{4} b^3 c^3 \text{PolyLog}\left[3,\frac{c+x}{x}\right]$$

Problem 152: Unable to integrate problem.

$$\int x \left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 135 leaves, 8 steps):

$$-\frac{3}{2} b c^2 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+\frac{3}{2} b c x \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2-\frac{1}{2} c^2 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3+$$
$$\frac{1}{2} x^2 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3-3 b^2 c^2 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{Log}\left[2-\frac{2}{1+\frac{c}{x}}\right]+\frac{3}{2} b^3 c^2\, \text{PolyLog}\left[2,\,-1+\frac{2}{1+\frac{c}{x}}\right]$$

Result (type 8, 897 leaves, 75 steps):

$$\frac{3}{4} a^2 b c x+\frac{3}{8} b^3\, \text{CannotIntegrate}\left[x\, \text{Log}\left[1-\frac{c}{x}\right]^2 \text{Log}\left[1+\frac{c}{x}\right],\, x\right]-\frac{3}{8} b^3\, \text{CannotIntegrate}\left[x\, \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]^2,\, x\right]-\frac{3}{4} a b^2 c x\, \text{Log}\left[1-\frac{c}{x}\right]+$$
$$\frac{3}{16} b c \left(1-\frac{c}{x}\right) x \left(2 a-b\, \text{Log}\left[1-\frac{c}{x}\right]\right)^2-\frac{1}{16} c^2 \left(2 a-b\, \text{Log}\left[1-\frac{c}{x}\right]\right)^3+\frac{1}{16} x^2 \left(2 a-b\, \text{Log}\left[1-\frac{c}{x}\right]\right)^3+\frac{3}{4} a b^2 c x\, \text{Log}\left[1+\frac{c}{x}\right]+\frac{3}{4} a^2 b x^2\, \text{Log}\left[1+\frac{c}{x}\right]-$$
$$\frac{3}{4} a b^2 x^2\, \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]+\frac{3}{4} a b^2 c^2\, \text{Log}[c-x]+\frac{3}{4} a b^2 c^2\, \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x]-\frac{3}{16} b c^2 \left(2 a-b\, \text{Log}\left[1-\frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c}{x}\right]+$$
$$\frac{3}{2} a b^2 c^2\, \text{Log}[x]+\frac{3}{4} a b^2 c^2\, \text{Log}[c-x]\, \text{Log}\left[\frac{x}{c}\right]-\frac{3}{4} a^2 b c^2\, \text{Log}[c+x]+\frac{3}{4} a b^2 c^2\, \text{Log}[c+x]+\frac{3}{4} a b^2 c^2\, \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x]-$$
$$\frac{3}{4} a b^2 c^2\, \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x]+\frac{3}{4} a b^2 c^2\, \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x]-\frac{3}{4} a b^2 c^2\, \text{Log}[c-x]\, \text{Log}\left[\frac{c+x}{2 c}\right]+\frac{3}{4} a b^2 c^2\, \text{Log}\left[\frac{c+x}{x}\right]+\frac{3}{4} a b^2 c x\, \text{Log}\left[\frac{c+x}{x}\right]-$$
$$\frac{3}{8} a b^2 c^2\, \text{Log}\left[\frac{c+x}{x}\right]^2+\frac{3}{16} b^3 c \left(1+\frac{c}{x}\right) x\, \text{Log}\left[\frac{c+x}{x}\right]^2+\frac{3}{8} a b^2 x^2\, \text{Log}\left[\frac{c+x}{x}\right]^2+\frac{3}{16} b^3 c^2\, \text{Log}\left[-\frac{c}{x}\right] \text{Log}\left[\frac{c+x}{x}\right]^2-\frac{1}{16} b^3 c^2\, \text{Log}\left[\frac{c+x}{x}\right]^3+$$
$$\frac{1}{16} b^3 x^2\, \text{Log}\left[\frac{c+x}{x}\right]^3+\frac{3}{8} b^2 c^2 \left(2 a-b\, \text{Log}\left[1-\frac{c}{x}\right]\right) \text{PolyLog}\left[2,\,1-\frac{c}{x}\right]-\frac{3}{4} a b^2 c^2\, \text{PolyLog}\left[2,\,\frac{c-x}{2 c}\right]-\frac{3}{4} a b^2 c^2\, \text{PolyLog}\left[2,\,-\frac{c}{x}\right]+$$
$$\frac{3}{8} b^3 c^2\, \text{PolyLog}\left[2,\,-\frac{c}{x}\right]-\frac{3}{8} b^3 c^2\, \text{PolyLog}\left[2,\,\frac{c}{x}\right]-\frac{3}{4} a b^2 c^2\, \text{PolyLog}\left[2,\,\frac{c+x}{2 c}\right]+\frac{3}{8} b^3 c^2\, \text{Log}\left[\frac{c+x}{x}\right] \text{PolyLog}\left[2,\,\frac{c+x}{x}\right]+$$
$$\frac{3}{4} a b^2 c^2\, \text{PolyLog}\left[2,\,1-\frac{x}{c}\right]+\frac{3}{4} a b^2 c^2\, \text{PolyLog}\left[2,\,1+\frac{x}{c}\right]+\frac{3}{8} b^3 c^2\, \text{PolyLog}\left[3,\,1-\frac{c}{x}\right]-\frac{3}{8} b^3 c^2\, \text{PolyLog}\left[3,\,\frac{c+x}{x}\right]$$

Problem 153: Unable to integrate problem.

$$\int \left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$c \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3+x \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3-3 b c \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \text{Log}\left[\frac{2 c}{c-x}\right]-$$
$$3 b^2 c \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{PolyLog}\left[2,\,1-\frac{2 c}{c-x}\right]+\frac{3}{2} b^3 c\, \text{PolyLog}\left[3,\,1-\frac{2 c}{c-x}\right]$$

Result (type 8, 642 leaves, 43 steps):

$$a^3 x+\frac{3}{8} b^3 \text{CannotIntegrate}\left[\text{Log}\left[1-\frac{c}{x}\right]^2 \text{Log}\left[1+\frac{c}{x}\right],x\right]-\frac{3}{8} b^3 \text{CannotIntegrate}\left[\text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]^2,x\right]-\frac{3}{2} a^2 b x \text{Log}\left[1-\frac{c}{x}\right]-$$
$$\frac{3}{4} a b^2 (c-x) \text{Log}\left[1-\frac{c}{x}\right]^2+\frac{1}{8} b^3 (c-x) \text{Log}\left[1-\frac{c}{x}\right]^3+\frac{3}{2} a^2 b x \text{Log}\left[1+\frac{c}{x}\right]-\frac{3}{2} a b^2 x \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]+\frac{3}{4} a b^2 (c+x) \text{Log}\left[1+\frac{c}{x}\right]^2+$$
$$\frac{1}{8} b^3 (c+x) \text{Log}\left[1+\frac{c}{x}\right]^3-\frac{3}{2} a b^2 c \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[-c-x]+\frac{3}{2} a^2 b c \text{Log}[c-x]+\frac{3}{2} a b^2 c \text{Log}[-c-x] \text{Log}\left[\frac{c-x}{2 c}\right]-\frac{3}{8} b^3 c \text{Log}\left[1-\frac{c}{x}\right]^2 \text{Log}\left[\frac{c}{x}\right]-$$
$$\frac{3}{2} a b^2 c \text{Log}[-c-x] \text{Log}\left[-\frac{x}{c}\right]+\frac{3}{2} a b^2 c \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[-c+x]+\frac{3}{2} a b^2 c \text{Log}\left[\frac{x}{c}\right] \text{Log}[-c+x]+\frac{3}{2} a^2 b c \text{Log}[c+x]-$$
$$\frac{3}{2} a b^2 c \text{Log}[-c+x] \text{Log}\left[\frac{c+x}{2 c}\right]-\frac{3}{8} b^3 c \text{Log}\left[-\frac{c}{x}\right] \text{Log}\left[\frac{c+x}{x}\right]^2-\frac{3}{4} b^3 c \text{Log}\left[1-\frac{c}{x}\right] \text{PolyLog}\left[2,1-\frac{c}{x}\right]-\frac{3}{2} a b^2 c \text{PolyLog}\left[2,\frac{c-x}{2 c}\right]+$$
$$\frac{3}{2} a b^2 c \text{PolyLog}\left[2,-\frac{c}{x}\right]-\frac{3}{2} a b^2 c \text{PolyLog}\left[2,\frac{c}{x}\right]+\frac{3}{2} a b^2 c \text{PolyLog}\left[2,\frac{c+x}{2 c}\right]-\frac{3}{4} b^3 c \text{Log}\left[\frac{c+x}{x}\right] \text{PolyLog}\left[2,\frac{c+x}{x}\right]+$$
$$\frac{3}{2} a b^2 c \text{PolyLog}\left[2,1-\frac{x}{c}\right]-\frac{3}{2} a b^2 c \text{PolyLog}\left[2,1+\frac{x}{c}\right]+\frac{3}{4} b^3 c \text{PolyLog}\left[3,1-\frac{c}{x}\right]+\frac{3}{4} b^3 c \text{PolyLog}\left[3,\frac{c+x}{x}\right]$$

Problem 155: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x^2} \, dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$-\frac{\left(a+b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{c}-\frac{\left(a+b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{x}+\frac{3 b \left(a+b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \text{Log}\left[\frac{2}{1-\frac{c}{x}}\right]}{c}+$$
$$\frac{3 b^2 \left(a+b \text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{PolyLog}\left[2,1-\frac{2}{1-\frac{c}{x}}\right]}{c}-\frac{3 b^3 \text{PolyLog}\left[3,1-\frac{2}{1-\frac{c}{x}}\right]}{2 c}$$

Result (type 4, 387 leaves, 82 steps):

$$\frac{\left(1-\frac{c}{x}\right) \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^3}{8 c}-\frac{3 b \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c+x}{2 x}\right]}{4 c}+\frac{3 b \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 c}-\frac{3 b \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 x}-$$
$$\frac{3 b^2 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right) \text{Log}\left[\frac{c+x}{x}\right]^2}{8 c}-\frac{3 b^2 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right) \text{Log}\left[\frac{c+x}{x}\right]^2}{8 x}-\frac{3 b^3 \text{Log}\left[-\frac{c-x}{2 x}\right] \text{Log}\left[\frac{c+x}{x}\right]^2}{4 c}-\frac{b^3 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]^3}{8 c}+$$
$$\frac{3 b^2 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right) \text{PolyLog}\left[2,-\frac{c-x}{2 x}\right]}{2 c}-\frac{3 b^3 \text{Log}\left[\frac{c+x}{x}\right] \text{PolyLog}\left[2,\frac{c+x}{2 x}\right]}{2 c}+\frac{3 b^3 \text{PolyLog}\left[3,-\frac{c-x}{2 x}\right]}{2 c}+\frac{3 b^3 \text{PolyLog}\left[3,\frac{c+x}{2 x}\right]}{2 c}$$

Problem 156: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x^3}\,dx$$

Optimal (type 4, 139 leaves, 9 steps):

$$-\frac{3 b \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2 c^2}-\frac{3 b \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2 c x}+\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{2 c^2}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{2 x^2}+\frac{3 b^2 \left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{Log}\left[\frac{2}{1-\frac{c}{x}}\right]}{c^2}+\frac{3 b^3\,\text{PolyLog}\left[2,1-\frac{2}{1-\frac{c}{x}}\right]}{2 c^2}$$

Result (type 8, 1098 leaves, 81 steps):

$$-\frac{3 b^3 \left(1-\frac{c}{x}\right)^2}{64 c^2}-\frac{3 a b^2 \left(1+\frac{c}{x}\right)^2}{16 c^2}+\frac{3 b^3 \left(1+\frac{c}{x}\right)^2}{64 c^2}+\frac{3 a^2 b}{8 x^2}+\frac{3 a b^2}{8 x^2}-\frac{3 a^2 b}{4 c x}-\frac{3 b^3}{2 c x}+\frac{3}{8} b^3\,\text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c}{x}\right]^2 \text{Log}\left[1+\frac{c}{x}\right]}{x^3},x\right]-$$
$$\frac{3}{8} b^3\,\text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]^2}{x^3},x\right]+\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right]}{8 c^2}-\frac{3 a b^2 \left(1-\frac{c}{x}\right) \text{Log}\left[1-\frac{c}{x}\right]}{4 c^2}-\frac{3 b^3 \left(1-\frac{c}{x}\right) \text{Log}\left[1-\frac{c}{x}\right]}{4 c^2}-$$
$$\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right]}{8 x^2}-\frac{3 b^2 \left(1-\frac{c}{x}\right)^2 \left(2 a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)}{32 c^2}+\frac{3 b \left(1-\frac{c}{x}\right) \left(2 a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{8 c^2}-\frac{3 b \left(1-\frac{c}{x}\right)^2 \left(2 a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{32 c^2}+$$
$$\frac{\left(1-\frac{c}{x}\right) \left(2 a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^3}{8 c^2}-\frac{\left(1-\frac{c}{x}\right)^2 \left(2 a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^3}{16 c^2}+\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]}{4 x^2}-\frac{3 a b^2 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x]}{4 c^2}-$$
$$\frac{3 a b^2 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right]}{4 c^2}-\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x]}{4 c^2}+\frac{3 a b^2 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x]}{4 c^2}-\frac{3 a b^2 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x]}{4 c^2}+$$
$$\frac{3 a b^2 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right]}{4 c^2}+\frac{3 a^2 b\,\text{Log}\left[\frac{c+x}{x}\right]}{4 c^2}+\frac{3 a b^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 c^2}-\frac{9 a b^2 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]}{4 c^2}+\frac{3 b^3 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]}{4 c^2}+$$
$$\frac{3 a b^2 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 c^2}-\frac{3 b^3 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]}{32 c^2}-\frac{3 a^2 b\,\text{Log}\left[\frac{c+x}{x}\right]}{4 x^2}-\frac{3 a b^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 x^2}+\frac{3 a b^2 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]^2}{4 c^2}-\frac{3 b^3 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]^2}{8 c^2}-$$
$$\frac{3 a b^2 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]^2}{8 c^2}+\frac{3 b^3 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]^2}{32 c^2}+\frac{b^3 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]^3}{8 c^2}-\frac{b^3 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]^3}{16 c^2}+\frac{3 a b^2\,\text{PolyLog}\left[2,\frac{c-x}{2 c}\right]}{4 c^2}+$$
$$\frac{3 a b^2\,\text{PolyLog}\left[2,-\frac{c}{x}\right]}{4 c^2}+\frac{3 a b^2\,\text{PolyLog}\left[2,\frac{c}{x}\right]}{4 c^2}+\frac{3 a b^2\,\text{PolyLog}\left[2,\frac{c+x}{2 c}\right]}{4 c^2}-\frac{3 a b^2\,\text{PolyLog}\left[2,1-\frac{x}{c}\right]}{4 c^2}-\frac{3 a b^2\,\text{PolyLog}\left[2,1+\frac{x}{c}\right]}{4 c^2}$$

Problem 171: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \left(a + b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2 dx$$

Optimal (type 3, 94 leaves, 9 steps):

$$\frac{1}{2} b c x^2 \left(a + b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right) - \frac{1}{4} c^2 \left(a + b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 + \frac{1}{4} x^4 \left(a + b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 + \frac{1}{4} b^2 c^2 \,\text{Log}\left[1 - \frac{c^2}{x^4}\right] + b^2 c^2 \,\text{Log}[x]$$

Result (type 4, 599 leaves, 59 steps):

$$\frac{1}{4} a b c x^2 - \frac{1}{8} b^2 c x^2 \,\text{Log}\left[1 - \frac{c}{x^2}\right] + \frac{1}{8} b c \left(1 - \frac{c}{x^2}\right) x^2 \left(2 a - b\,\text{Log}\left[1 - \frac{c}{x^2}\right]\right) - \frac{1}{16} c^2 \left(2 a - b\,\text{Log}\left[1 - \frac{c}{x^2}\right]\right)^2 + \frac{1}{16} x^4 \left(2 a - b\,\text{Log}\left[1 - \frac{c}{x^2}\right]\right)^2 +$$
$$\frac{1}{8} b^2 c^2 \,\text{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{4} b^2 c x^2 \,\text{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{4} a b x^4 \,\text{Log}\left[1 + \frac{c}{x^2}\right] - \frac{1}{8} b^2 x^4 \,\text{Log}\left[1 - \frac{c}{x^2}\right] \text{Log}\left[1 + \frac{c}{x^2}\right] - \frac{1}{16} b^2 c^2 \,\text{Log}\left[1 + \frac{c}{x^2}\right]^2 +$$
$$\frac{1}{16} b^2 x^4 \,\text{Log}\left[1 + \frac{c}{x^2}\right]^2 + \frac{1}{2} a b c^2 \,\text{Log}[x] + \frac{1}{2} b^2 c^2 \,\text{Log}[x] + \frac{1}{8} b^2 c^2 \,\text{Log}\left[c - x^2\right] + \frac{1}{8} b^2 c^2 \,\text{Log}\left[1 + \frac{c}{x^2}\right] \text{Log}\left[c - x^2\right] +$$
$$\frac{1}{8} b^2 c^2 \,\text{Log}\left[\frac{x^2}{c}\right] \text{Log}\left[c - x^2\right] - \frac{1}{4} a b c^2 \,\text{Log}\left[c + x^2\right] + \frac{1}{8} b^2 c^2 \,\text{Log}\left[c + x^2\right] + \frac{1}{8} b^2 c^2 \,\text{Log}\left[1 - \frac{c}{x^2}\right] \text{Log}\left[c + x^2\right] + \frac{1}{8} b^2 c^2 \,\text{Log}\left[-\frac{x^2}{c}\right] \text{Log}\left[c + x^2\right] -$$
$$\frac{1}{8} b^2 c^2 \,\text{Log}\left[\frac{c - x^2}{2 c}\right] \text{Log}\left[c + x^2\right] - \frac{1}{8} b^2 c^2 \,\text{Log}\left[c - x^2\right] \text{Log}\left[\frac{c + x^2}{2 c}\right] - \frac{1}{8} b^2 c^2 \,\text{PolyLog}\left[2, -\frac{c}{x^2}\right] - \frac{1}{8} b^2 c^2 \,\text{PolyLog}\left[2, \frac{c}{x^2}\right] -$$
$$\frac{1}{8} b^2 c^2 \,\text{PolyLog}\left[2, \frac{c - x^2}{2 c}\right] - \frac{1}{8} b^2 c^2 \,\text{PolyLog}\left[2, \frac{c + x^2}{2 c}\right] + \frac{1}{8} b^2 c^2 \,\text{PolyLog}\left[2, \frac{c + x^2}{c}\right] + \frac{1}{8} b^2 c^2 \,\text{PolyLog}\left[2, 1 - \frac{x^2}{c}\right]$$

Problem 172: Result valid but suboptimal antiderivative.

$$\int x \left(a + b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2 dx$$

Optimal (type 4, 94 leaves, 5 steps):

$$-\frac{1}{2} c \left(a + b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 + \frac{1}{2} x^2 \left(a + b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 - b c \left(a + b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right) \text{Log}\left[2 - \frac{2}{1 + \frac{c}{x^2}}\right] + \frac{1}{2} b^2 c \,\text{PolyLog}\left[2, -1 + \frac{2}{1 + \frac{c}{x^2}}\right]$$

Result (type 4, 404 leaves, 34 steps):

$$\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2+\frac{1}{2}a\,b\,x^2\,\text{Log}\left[1+\frac{c}{x^2}\right]-\frac{1}{4}b^2x^2\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[1+\frac{c}{x^2}\right]+\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\,\text{Log}\left[1+\frac{c}{x^2}\right]^2+a\,b\,c\,\text{Log}[x]-$$
$$\frac{1}{4}b^2c\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[-c-x^2\right]-\frac{1}{4}b^2c\,\text{Log}\left[-\frac{x^2}{c}\right]\text{Log}\left[-c-x^2\right]+\frac{1}{4}b^2c\,\text{Log}\left[-c-x^2\right]\text{Log}\left[\frac{c-x^2}{2c}\right]+\frac{1}{4}b^2c\,\text{Log}\left[1+\frac{c}{x^2}\right]\text{Log}\left[-c+x^2\right]+$$
$$\frac{1}{4}b^2c\,\text{Log}\left[\frac{x^2}{c}\right]\text{Log}\left[-c+x^2\right]+\frac{1}{2}a\,b\,c\,\text{Log}\left[c+x^2\right]-\frac{1}{4}b^2c\,\text{Log}\left[-c+x^2\right]\text{Log}\left[\frac{c+x^2}{2c}\right]+\frac{1}{4}b^2c\,\text{PolyLog}\left[2,-\frac{c}{x^2}\right]-$$
$$\frac{1}{4}b^2c\,\text{PolyLog}\left[2,\frac{c}{x^2}\right]-\frac{1}{4}b^2c\,\text{PolyLog}\left[2,\frac{c-x^2}{2c}\right]+\frac{1}{4}b^2c\,\text{PolyLog}\left[2,\frac{c+x^2}{2c}\right]-\frac{1}{4}b^2c\,\text{PolyLog}\left[2,\frac{c+x^2}{c}\right]+\frac{1}{4}b^2c\,\text{PolyLog}\left[2,1-\frac{x^2}{c}\right]$$

Problem 174: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^3}\,dx$$

Optimal (type 4, 99 leaves, 6 steps):

$$-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{2c}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{2x^2}+\frac{b\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)\text{Log}\left[\frac{2}{1-\frac{c}{x^2}}\right]}{c}+\frac{b^2\,\text{PolyLog}\left[2,1-\frac{2}{1-\frac{c}{x^2}}\right]}{2c}$$

Result (type 4, 207 leaves, 28 steps):

$$\frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2}{8c}-\frac{b\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)\text{Log}\left[\frac{1}{2}\left(1+\frac{c}{x^2}\right)\right]}{4c}-\frac{b^2\,\text{Log}\left[\frac{1}{2}\left(1-\frac{c}{x^2}\right)\right]\text{Log}\left[1+\frac{c}{x^2}\right]}{4c}-$$
$$\frac{b\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)\text{Log}\left[1+\frac{c}{x^2}\right]}{4x^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[1+\frac{c}{x^2}\right]^2}{8c}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-\frac{c}{x^2}\right)\right]}{4c}-\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+\frac{c}{x^2}\right)\right]}{4c}$$

Problem 175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^5}\,dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{a\,b}{2c\,x^2}-\frac{b^2\,\text{ArcCoth}\left[\frac{x^2}{c}\right]}{2c\,x^2}+\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{4c^2}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{4x^4}-\frac{b^2\,\text{Log}\left[1-\frac{c^2}{x^4}\right]}{4c^2}$$

Result (type 4, 770 leaves, 67 steps):

$$-\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2}+\frac{ab}{8x^4}+\frac{b^2}{16x^4}-\frac{3ab}{4cx^2}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]}{16c^2}-\frac{3b^2\left(1-\frac{c}{x^2}\right)\text{Log}\left[1-\frac{c}{x^2}\right]}{8c^2}-\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]}{16x^4}-$$
$$\frac{b\left(1-\frac{c}{x^2}\right)^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)}{16c^2}+\frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2}{8c^2}-\frac{\left(1-\frac{c}{x^2}\right)^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[1+\frac{c}{x^2}\right]}{4c^2}+$$
$$\frac{b^2\left(1+\frac{c}{x^2}\right)^2\text{Log}\left[1+\frac{c}{x^2}\right]}{16c^2}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[1+\frac{c}{x^2}\right]}{8x^4}+\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[1+\frac{c}{x^2}\right]^2}{8c^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)^2\text{Log}\left[1+\frac{c}{x^2}\right]^2}{16c^2}-$$
$$\frac{b^2\,\text{Log}\left[1+\frac{c}{x^2}\right]\text{Log}\left[c-x^2\right]}{8c^2}-\frac{b^2\,\text{Log}\left[\frac{x^2}{c}\right]\text{Log}\left[c-x^2\right]}{8c^2}-\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[c+x^2\right]}{8c^2}-\frac{b^2\,\text{Log}\left[-\frac{x^2}{c}\right]\text{Log}\left[c+x^2\right]}{8c^2}+\frac{b^2\,\text{Log}\left[\frac{c-x^2}{2c}\right]\text{Log}\left[c+x^2\right]}{8c^2}+$$
$$\frac{b^2\,\text{Log}\left[c-x^2\right]\text{Log}\left[\frac{c+x^2}{2c}\right]}{8c^2}+\frac{ab\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{4c^2}+\frac{b^2\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{16c^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[\frac{c+x^2}{x^2}\right]}{8c^2}-\frac{ab\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{4x^4}-\frac{b^2\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{16x^4}+$$
$$\frac{b^2\,\text{PolyLog}\left[2,-\frac{c}{x^2}\right]}{8c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c}{x^2}\right]}{8c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c-x^2}{2c}\right]}{8c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c+x^2}{2c}\right]}{8c^2}-\frac{b^2\,\text{PolyLog}\left[2,\frac{c+x^2}{c}\right]}{8c^2}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{x^2}{c}\right]}{8c^2}$$

Problem 184: Result optimal but 1 more steps used.

$$\int (d\,x)^m\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)dx$$

Optimal (type 5, 75 leaves, 3 steps):

$$\frac{(d\,x)^{1+m}\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)}{d\,(1+m)}-\frac{2\,b\,c\,d\,(d\,x)^{-1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1-m}{4},\frac{5-m}{4},\frac{c^2}{x^4}\right]}{1-m^2}$$

Result (type 5, 75 leaves, 4 steps):

$$\frac{(d\,x)^{1+m}\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)}{d\,(1+m)}-\frac{2\,b\,c\,d\,(d\,x)^{-1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1-m}{4},\frac{5-m}{4},\frac{c^2}{x^4}\right]}{1-m^2}$$

Problem 195: Unable to integrate problem.

$$\int x^3\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2dx$$

Optimal (type 3, 211 leaves, 22 steps):

$$\frac{a\,b\,\sqrt{x}}{2\,c^7}+\frac{71\,b^2\,x}{420\,c^6}+\frac{3\,b^2\,x^2}{70\,c^4}+\frac{b^2\,x^3}{84\,c^2}+\frac{b^2\,\sqrt{x}\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{2\,c^7}+\frac{b\,x^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{6\,c^5}+\frac{b\,x^{5/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{10\,c^3}+\frac{b\,x^{7/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{14\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{4\,c^8}+\frac{1}{4}\,x^4\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2+\frac{44\,b^2\,\text{Log}\left[1-c^2\,x\right]}{105\,c^8}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^3\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2,\,x\right]$$

Problem 196: Unable to integrate problem.

$$\int x^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2\,dx$$

Optimal (type 3, 173 leaves, 17 steps):

$$\frac{2\,a\,b\,\sqrt{x}}{3\,c^5}+\frac{8\,b^2\,x}{45\,c^4}+\frac{b^2\,x^2}{30\,c^2}+\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{3\,c^5}+\frac{2\,b\,x^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{9\,c^3}+\frac{2\,b\,x^{5/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{15\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{3\,c^6}+\frac{1}{3}\,x^3\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2+\frac{23\,b^2\,\text{Log}\left[1-c^2\,x\right]}{45\,c^6}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2,\,x\right]$$

Problem 197: Unable to integrate problem.

$$\int x\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2\,dx$$

Optimal (type 3, 129 leaves, 12 steps):

$$\frac{a\,b\,\sqrt{x}}{c^3}+\frac{b^2\,x}{6\,c^2}+\frac{b^2\,\sqrt{x}\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{c^3}+\frac{b\,x^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{3\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{2\,c^4}+\frac{1}{2}\,x^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2+\frac{2\,b^2\,\text{Log}\left[1-c^2\,x\right]}{3\,c^4}$$

Result (type 8, 18 leaves, 0 steps):

$$\text{Unintegrable}\left[x\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2,\,x\right]$$

Problem 198: Unable to integrate problem.

$$\int \left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2 \,dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{2\,a\,b\,\sqrt{x}}{c} + \frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}{c} - \frac{\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{c^2} + x\,\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2 + \frac{b^2\,\text{Log}\left[1 - c^2\,x\right]}{c^2}$$

Result (type 8, 16 leaves, 0 steps):

$$\text{Unintegrable}\left[\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2,\,x\right]$$

Problem 200: Unable to integrate problem.

$$\int \frac{\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{x^2} \,dx$$

Optimal (type 3, 85 leaves, 9 steps):

$$-\frac{2\,b\,c\,\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)}{\sqrt{x}} + c^2\,\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2 - \frac{\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{x} + b^2\,c^2\,\text{Log}[x] - b^2\,c^2\,\text{Log}\left[1 - c^2\,x\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{x^2},\,x\right]$$

Problem 201: Unable to integrate problem.

$$\int \frac{\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{x^3} \,dx$$

Optimal (type 3, 133 leaves, 14 steps):

$$-\frac{b^2\,c^2}{6\,x} - \frac{b\,c\,\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)}{3\,x^{3/2}} - \frac{b\,c^3\,\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)}{\sqrt{x}} +$$

$$\frac{1}{2}\,c^4\,\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2 - \frac{\left(a + b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{2\,x^2} + \frac{2}{3}\,b^2\,c^4\,\text{Log}[x] - \frac{2}{3}\,b^2\,c^4\,\text{Log}\left[1 - c^2\,x\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{x^3},x\right]$$

Problem 202: Unable to integrate problem.

$$\int x^3\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^3\,dx$$

Optimal (type 4, 374 leaves, 54 steps):

$$\frac{47 b^3\sqrt{x}}{70 c^7}+\frac{23 b^3 x^{3/2}}{420 c^5}+\frac{b^3 x^{5/2}}{140 c^3}-\frac{47 b^3\,\text{ArcTanh}\left[c\sqrt{x}\right]}{70 c^8}+\frac{71 b^2 x\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{140 c^6}+\frac{9 b^2 x^2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{70 c^4}+\frac{b^2 x^3\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{28 c^2}+\frac{44 b\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{35 c^8}+\frac{3 b\sqrt{x}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{4 c^7}+\frac{b x^{3/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{4 c^5}+\frac{3 b x^{5/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{20 c^3}+\frac{3 b x^{7/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{28 c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^3}{4 c^8}+\frac{1}{4}x^4\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^3-\frac{88 b^2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2}{1-c\sqrt{x}}\right]}{35 c^8}-\frac{44 b^3\,\text{PolyLog}\left[2,1-\frac{2}{1-c\sqrt{x}}\right]}{35 c^8}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^3\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^3,x\right]$$

Problem 203: Unable to integrate problem.

$$\int x^2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^3\,dx$$

Optimal (type 4, 304 leaves, 34 steps):

$$\frac{19 b^3\sqrt{x}}{30 c^5}+\frac{b^3 x^{3/2}}{30 c^3}-\frac{19 b^3\,\text{ArcTanh}\left[c\sqrt{x}\right]}{30 c^6}+\frac{8 b^2 x\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{15 c^4}+\frac{b^2 x^2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{10 c^2}+\frac{23 b\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{15 c^6}+\frac{b\sqrt{x}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{c^5}+\frac{b x^{3/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{3 c^3}+\frac{b x^{5/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{5 c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^3}{3 c^6}+\frac{1}{3}x^3\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^3-\frac{46 b^2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2}{1-c\sqrt{x}}\right]}{15 c^6}-\frac{23 b^3\,\text{PolyLog}\left[2,1-\frac{2}{1-c\sqrt{x}}\right]}{15 c^6}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^2 \left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3,\,x\right]$$

Problem 204: Unable to integrate problem.

$$\int x \left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 234 leaves, 19 steps):

$$\frac{b^3\sqrt{x}}{2\,c^3}-\frac{b^3\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}{2\,c^4}+\frac{b^2\,x\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)}{2\,c^2}+\frac{2\,b\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{c^4}+\frac{3\,b\,\sqrt{x}\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{2\,c^3}+\frac{b\,x^{3/2}\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{2\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3}{2\,c^4}+\frac{1}{2}\,x^2\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3-\frac{4\,b^2\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)\,\text{Log}\left[\frac{2}{1-c\,\sqrt{x}}\right]}{c^4}-\frac{2\,b^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,\sqrt{x}}\right]}{c^4}$$

Result (type 8, 18 leaves, 0 steps):

$$\text{Unintegrable}\left[x \left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3,\,x\right]$$

Problem 205: Unable to integrate problem.

$$\int \left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{3\,b\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{c^2}+\frac{3\,b\,\sqrt{x}\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^2}{c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3}{c^2}+x\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3-\frac{6\,b^2\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)\,\text{Log}\left[\frac{2}{1-c\,\sqrt{x}}\right]}{c^2}-\frac{3\,b^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,\sqrt{x}}\right]}{c^2}$$

Result (type 8, 16 leaves, 0 steps):

$$\text{Unintegrable}\left[\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3,\,x\right]$$

Problem 207: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x^2}\,dx$$

Optimal (type 4, 142 leaves, 8 steps):

$$3\,b\,c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2-\frac{3\,b\,c\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{\sqrt{x}}+c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x}+6\,b^2\,c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[2-\frac{2}{1+c\,\sqrt{x}}\right]-3\,b^3\,c^2\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+c\,\sqrt{x}}\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x^2},\,x\right]$$

Problem 208: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x^3}\,dx$$

Optimal (type 4, 234 leaves, 17 steps):

$$-\frac{b^3\,c^3}{2\,\sqrt{x}}+\frac{1}{2}\,b^3\,c^4\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]-\frac{b^2\,c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{2\,x}+2\,b\,c^4\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2-\frac{b\,c\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{2\,x^{3/2}}-\frac{3\,b\,c^3\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{2\,\sqrt{x}}+\frac{1}{2}\,c^4\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{2\,x^2}+4\,b^2\,c^4\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[2-\frac{2}{1+c\,\sqrt{x}}\right]-2\,b^3\,c^4\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+c\,\sqrt{x}}\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x^3},\,x\right]$$

Problem 221: Unable to integrate problem.

$$\int x^2 \left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2 dx$$

Optimal (type 3, 101 leaves, 7 steps):

$$\frac{2\, a\, b\, x^{3/2}}{3\, c} + \frac{2\, b^2\, x^{3/2} \operatorname{ArcTanh}\left[c\, x^{3/2}\right]}{3\, c} - \frac{\left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2}{3\, c^2} + \frac{1}{3} x^3 \left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2 + \frac{b^2 \operatorname{Log}\left[1 - c^2\, x^3\right]}{3\, c^2}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^2 \left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2, x\right]$$

Problem 223: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2}{x^4} dx$$

Optimal (type 3, 96 leaves, 9 steps):

$$-\frac{2\, b\, c \left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)}{3\, x^{3/2}} + \frac{1}{3} c^2 \left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2 - \frac{\left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2}{3\, x^3} + b^2\, c^2 \operatorname{Log}[x] - \frac{1}{3} b^2\, c^2 \operatorname{Log}\left[1 - c^2\, x^3\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a + b \operatorname{ArcTanh}\left[c\, x^{3/2}\right]\right)^2}{x^4}, x\right]$$

Test results for the 49 problems in "7.3.3 (d+e x)^m (a+b arctanh(c x^n))^p.m"

Problem 23: Result valid but suboptimal antiderivative.

$$\int (d + e\, x)^3 \left(a + b \operatorname{ArcTanh}\left[c\, x^2\right]\right) dx$$

Optimal (type 3, 182 leaves, 13 steps):

$$\frac{2\, b\, d\, e^2\, x}{c} + \frac{b\, e^3\, x^2}{4\, c} + \frac{b\, d \left(c\, d^2 - e^2\right) \operatorname{ArcTan}\left[\sqrt{c}\, x\right]}{c^{3/2}} - \frac{b\, d \left(c\, d^2 + e^2\right) \operatorname{ArcTanh}\left[\sqrt{c}\, x\right]}{c^{3/2}} +$$
$$\frac{(d + e\, x)^4 \left(a + b \operatorname{ArcTanh}\left[c\, x^2\right]\right)}{4\, e} + \frac{b \left(c^2\, d^4 + 6\, c\, d^2\, e^2 + e^4\right) \operatorname{Log}\left[1 - c\, x^2\right]}{8\, c^2\, e} - \frac{b \left(c^2\, d^4 - 6\, c\, d^2\, e^2 + e^4\right) \operatorname{Log}\left[1 + c\, x^2\right]}{8\, c^2\, e}$$

Result (type 3, 220 leaves, 19 steps):

$$\frac{2\,b\,d\,e^2\,x}{c}+\frac{b\,e^3\,x^2}{4\,c}+\frac{a\,(d+e\,x)^4}{4\,e}+\frac{b\,d^3\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{\sqrt{c}}-\frac{b\,d\,e^2\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{c^{3/2}}-\frac{b\,d^3\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{\sqrt{c}}-\frac{b\,d\,e^2\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{c^{3/2}}-\frac{b\,e^3\,\text{ArcTanh}\left[c\,x^2\right]}{4\,c^2}+$$
$$b\,d^3\,x\,\text{ArcTanh}\left[c\,x^2\right]+\frac{3}{2}\,b\,d^2\,e\,x^2\,\text{ArcTanh}\left[c\,x^2\right]+b\,d\,e^2\,x^3\,\text{ArcTanh}\left[c\,x^2\right]+\frac{1}{4}\,b\,e^3\,x^4\,\text{ArcTanh}\left[c\,x^2\right]+\frac{3\,b\,d^2\,e\,\text{Log}\left[1-c^2\,x^4\right]}{4\,c}$$

Problem 24: Result optimal but 1 more steps used.

$$\int (d+e\,x)^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,dx$$

Optimal (type 3, 158 leaves, 11 steps):

$$\frac{2\,b\,e^2\,x}{3\,c}+\frac{b\,\left(3\,c\,d^2-e^2\right)\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{3\,c^{3/2}}-\frac{b\,\left(3\,c\,d^2+e^2\right)\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{3\,c^{3/2}}+$$
$$\frac{(d+e\,x)^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,e}+\frac{b\,d\,\left(c\,d^2+3\,e^2\right)\,\text{Log}\left[1-c\,x^2\right]}{6\,c\,e}-\frac{b\,d\,\left(c\,d^2-3\,e^2\right)\,\text{Log}\left[1+c\,x^2\right]}{6\,c\,e}$$

Result (type 3, 158 leaves, 12 steps):

$$\frac{2\,b\,e^2\,x}{3\,c}+\frac{b\,\left(3\,c\,d^2-e^2\right)\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{3\,c^{3/2}}-\frac{b\,\left(3\,c\,d^2+e^2\right)\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{3\,c^{3/2}}+$$
$$\frac{(d+e\,x)^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,e}+\frac{b\,d\,\left(c\,d^2+3\,e^2\right)\,\text{Log}\left[1-c\,x^2\right]}{6\,c\,e}-\frac{b\,d\,\left(c\,d^2-3\,e^2\right)\,\text{Log}\left[1+c\,x^2\right]}{6\,c\,e}$$

Problem 26: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{d+e\,x}\,dx$$

Optimal (type 4, 325 leaves, 19 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,\text{Log}[d+e\,x]}{e}-\frac{b\,\text{Log}\left[\frac{e\left(1-\sqrt{-c}\,x\right)}{\sqrt{-c}\,d+e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{Log}\left[-\frac{e\left(1+\sqrt{-c}\,x\right)}{\sqrt{-c}\,d-e}\right]\,\text{Log}[d+e\,x]}{2\,e}+\frac{b\,\text{Log}\left[\frac{e\left(1-\sqrt{c}\,x\right)}{\sqrt{c}\,d+e}\right]\,\text{Log}[d+e\,x]}{2\,e}+$$
$$\frac{b\,\text{Log}\left[-\frac{e\left(1+\sqrt{c}\,x\right)}{\sqrt{c}\,d-e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{-c}\,(d+e\,x)}{\sqrt{-c}\,d-e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-e}\right]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{-c}\,(d+e\,x)}{\sqrt{-c}\,d+e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+e}\right]}{2\,e}$$

Result (type 8, 30 leaves, 2 steps):

$$b\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^2\right]}{d+e\,x},\,x\right]+\frac{a\,\text{Log}[d+e\,x]}{e}$$

Problem 27: Result optimal but 1 more steps used.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d+e\,x)^2}\,dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\frac{b\sqrt{c}\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{c\,d^2+e^2}-\frac{b\sqrt{c}\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{c\,d^2-e^2}-\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{e\,(d+e\,x)}+\frac{2\,b\,c\,d\,e\,\text{Log}[d+e\,x]}{c^2\,d^4-e^4}-\frac{b\,c\,d\,\text{Log}\left[1-c\,x^2\right]}{2\,e\,\left(c\,d^2-e^2\right)}+\frac{b\,c\,d\,\text{Log}\left[1+c\,x^2\right]}{2\,e\,\left(c\,d^2+e^2\right)}$$

Result (type 3, 166 leaves, 10 steps):

$$\frac{b\sqrt{c}\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{c\,d^2+e^2}-\frac{b\sqrt{c}\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{c\,d^2-e^2}-\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{e\,(d+e\,x)}+\frac{2\,b\,c\,d\,e\,\text{Log}[d+e\,x]}{c^2\,d^4-e^4}-\frac{b\,c\,d\,\text{Log}\left[1-c\,x^2\right]}{2\,e\,\left(c\,d^2-e^2\right)}+\frac{b\,c\,d\,\text{Log}\left[1+c\,x^2\right]}{2\,e\,\left(c\,d^2+e^2\right)}$$

Problem 28: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d+e\,x)^3}\,dx$$

Optimal (type 3, 226 leaves, 9 steps):

$$-\frac{b\,c\,d\,e}{\left(c^2\,d^4-e^4\right)\,(d+e\,x)}+\frac{b\,c^{3/2}\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{\left(c\,d^2+e^2\right)^2}-\frac{b\,c^{3/2}\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{\left(c\,d^2-e^2\right)^2}-$$
$$\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{2\,e\,(d+e\,x)^2}+\frac{b\,c\,e\,\left(3\,c^2\,d^4+e^4\right)\,\text{Log}[d+e\,x]}{\left(c^2\,d^4-e^4\right)^2}-\frac{b\,c\,\left(c\,d^2+e^2\right)\,\text{Log}\left[1-c\,x^2\right]}{4\,e\,\left(c\,d^2-e^2\right)^2}+\frac{b\,c\,\left(c\,d^2-e^2\right)\,\text{Log}\left[1+c\,x^2\right]}{4\,e\,\left(c\,d^2+e^2\right)^2}$$

Result (type 8, 34 leaves, 2 steps):

$$-\frac{a}{2\,e\,(d+e\,x)^2}+b\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^2\right]}{(d+e\,x)^3},\,x\right]$$

Problem 29: Result valid but suboptimal antiderivative.

$$\int (d+e\,x)\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2\,dx$$

Optimal (type 4, 1085 leaves, 77 steps):

$$a^2 d x+\frac{2 a b d \text{ArcTan}\left[\sqrt{c}\ x\right]}{\sqrt{c}}+\frac{i b^2 d \text{ArcTan}\left[\sqrt{c}\ x\right]^2}{\sqrt{c}}-\frac{2 a b d \text{ArcTanh}\left[\sqrt{c}\ x\right]}{\sqrt{c}}-\frac{b^2 d \text{ArcTanh}\left[\sqrt{c}\ x\right]^2}{\sqrt{c}}+$$

$$\frac{e \left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2}{2 c}+\frac{1}{2} e x^2 \left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2+\frac{2 b^2 d \text{ArcTanh}\left[\sqrt{c}\ x\right] \text{Log}\left[\frac{2}{1-\sqrt{c}\ x}\right]}{\sqrt{c}}-\frac{2 b^2 d \text{ArcTan}\left[\sqrt{c}\ x\right] \text{Log}\left[\frac{2}{1-i \sqrt{c}\ x}\right]}{\sqrt{c}}+$$

$$\frac{b^2 d \text{ArcTan}\left[\sqrt{c}\ x\right] \text{Log}\left[\frac{(1+i) \left(1-\sqrt{c}\ x\right)}{1-i \sqrt{c}\ x}\right]}{\sqrt{c}}+\frac{2 b^2 d \text{ArcTan}\left[\sqrt{c}\ x\right] \text{Log}\left[\frac{2}{1+i \sqrt{c}\ x}\right]}{\sqrt{c}}-\frac{2 b^2 d \text{ArcTanh}\left[\sqrt{c}\ x\right] \text{Log}\left[\frac{2}{1+\sqrt{c}\ x}\right]}{\sqrt{c}}+$$

$$\frac{b^2 d \text{ArcTanh}\left[\sqrt{c}\ x\right] \text{Log}\left[-\frac{2 \sqrt{c} \left(1-\sqrt{-c}\ x\right)}{\left(\sqrt{-c}-\sqrt{c}\right) \left(1+\sqrt{c}\ x\right)}\right]}{\sqrt{c}}+\frac{b^2 d \text{ArcTanh}\left[\sqrt{c}\ x\right] \text{Log}\left[\frac{2 \sqrt{c} \left(1+\sqrt{-c}\ x\right)}{\left(\sqrt{-c}+\sqrt{c}\right) \left(1+\sqrt{c}\ x\right)}\right]}{\sqrt{c}}+\frac{b^2 d \text{ArcTan}\left[\sqrt{c}\ x\right] \text{Log}\left[\frac{(1-i) \left(1+\sqrt{c}\ x\right)}{1-i \sqrt{c}\ x}\right]}{\sqrt{c}}-$$

$$\frac{b e \left(a+b \text{ArcTanh}\left[c x^2\right]\right) \text{Log}\left[\frac{2}{1-c x^2}\right]}{c}-a b d x \text{Log}\left[1-c x^2\right]-\frac{b^2 d \text{ArcTan}\left[\sqrt{c}\ x\right] \text{Log}\left[1-c x^2\right]}{\sqrt{c}}+\frac{b^2 d \text{ArcTanh}\left[\sqrt{c}\ x\right] \text{Log}\left[1-c x^2\right]}{\sqrt{c}}+$$

$$\frac{1}{4} b^2 d x \text{Log}\left[1-c x^2\right]^2+a b d x \text{Log}\left[1+c x^2\right]+\frac{b^2 d \text{ArcTan}\left[\sqrt{c}\ x\right] \text{Log}\left[1+c x^2\right]}{\sqrt{c}}-\frac{b^2 d \text{ArcTanh}\left[\sqrt{c}\ x\right] \text{Log}\left[1+c x^2\right]}{\sqrt{c}}-$$

$$\frac{1}{2} b^2 d x \text{Log}\left[1-c x^2\right] \text{Log}\left[1+c x^2\right]+\frac{1}{4} b^2 d x \text{Log}\left[1+c x^2\right]^2+\frac{b^2 d \text{PolyLog}\left[2,1-\frac{2}{1-\sqrt{c}\ x}\right]}{\sqrt{c}}+\frac{i b^2 d \text{PolyLog}\left[2,1-\frac{2}{1-i \sqrt{c}\ x}\right]}{\sqrt{c}}-$$

$$\frac{i b^2 d \text{PolyLog}\left[2,1-\frac{(1+i) \left(1-\sqrt{c}\ x\right)}{1-i \sqrt{c}\ x}\right]}{2 \sqrt{c}}+\frac{i b^2 d \text{PolyLog}\left[2,1-\frac{2}{1+i \sqrt{c}\ x}\right]}{\sqrt{c}}+\frac{b^2 d \text{PolyLog}\left[2,1-\frac{2}{1+\sqrt{c}\ x}\right]}{\sqrt{c}}-\frac{b^2 d \text{PolyLog}\left[2,1+\frac{2 \sqrt{c} \left(1-\sqrt{-c}\ x\right)}{\left(\sqrt{-c}-\sqrt{c}\right) \left(1+\sqrt{c}\ x\right)}\right]}{2 \sqrt{c}}-$$

$$\frac{b^2 d \text{PolyLog}\left[2,1-\frac{2 \sqrt{c} \left(1+\sqrt{-c}\ x\right)}{\left(\sqrt{-c}+\sqrt{c}\right) \left(1+\sqrt{c}\ x\right)}\right]}{2 \sqrt{c}}-\frac{i b^2 d \text{PolyLog}\left[2,1-\frac{(1-i) \left(1+\sqrt{c}\ x\right)}{1-i \sqrt{c}\ x}\right]}{2 \sqrt{c}}-\frac{b^2 e \text{PolyLog}\left[2,1-\frac{2}{1-c x^2}\right]}{2 c}$$

Result (type 4, 1216 leaves, 104 steps):

$$\frac{a^2\,(d+e\,x)^2}{2\,e}+\frac{2\,a\,b\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{\sqrt{c}}+\frac{i\,b^2\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]^2}{\sqrt{c}}-\frac{2\,a\,b\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{\sqrt{c}}-\frac{b^2\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]^2}{\sqrt{c}}+2\,a\,b\,d\,x\,\text{ArcTanh}\left[c\,x^2\right]+$$

$$a\,b\,e\,x^2\,\text{ArcTanh}\left[c\,x^2\right]+\frac{2\,b^2\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]\,\text{Log}\left[\frac{2}{1-\sqrt{c}\,x}\right]}{\sqrt{c}}-\frac{2\,b^2\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]\,\text{Log}\left[\frac{2}{1-i\,\sqrt{c}\,x}\right]}{\sqrt{c}}+\frac{b^2\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]\,\text{Log}\left[\frac{(1+i)\,\left(1-\sqrt{c}\,x\right)}{1-i\,\sqrt{c}\,x}\right]}{\sqrt{c}}+$$

$$\frac{2\,b^2\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]\,\text{Log}\left[\frac{2}{1+i\,\sqrt{c}\,x}\right]}{\sqrt{c}}-\frac{2\,b^2\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]\,\text{Log}\left[\frac{2}{1+\sqrt{c}\,x}\right]}{\sqrt{c}}+\frac{b^2\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]\,\text{Log}\left[-\frac{2\,\sqrt{c}\,\left(1-\sqrt{-c}\,x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\,\left(1+\sqrt{c}\,x\right)}\right]}{\sqrt{c}}+$$

$$\frac{b^2\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]\,\text{Log}\left[\frac{2\,\sqrt{c}\,\left(1+\sqrt{-c}\,x\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\,\left(1+\sqrt{c}\,x\right)}\right]}{\sqrt{c}}+\frac{b^2\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]\,\text{Log}\left[\frac{(1-i)\,\left(1+\sqrt{c}\,x\right)}{1-i\,\sqrt{c}\,x}\right]}{\sqrt{c}}-\frac{b^2\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]\,\text{Log}\left[1-c\,x^2\right]}{\sqrt{c}}+$$

$$\frac{b^2\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]\,\text{Log}\left[1-c\,x^2\right]}{\sqrt{c}}+\frac{1}{4}\,b^2\,d\,x\,\text{Log}\left[1-c\,x^2\right]^2-\frac{b^2\,e\,\left(1-c\,x^2\right)\,\text{Log}\left[1-c\,x^2\right]^2}{8\,c}-\frac{b^2\,e\,\text{Log}\left[1-c\,x^2\right]\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{4\,c}+$$

$$\frac{b^2\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]\,\text{Log}\left[1+c\,x^2\right]}{\sqrt{c}}-\frac{b^2\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]\,\text{Log}\left[1+c\,x^2\right]}{\sqrt{c}}+\frac{b^2\,e\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\text{Log}\left[1+c\,x^2\right]}{4\,c}-$$

$$\frac{1}{2}\,b^2\,d\,x\,\text{Log}\left[1-c\,x^2\right]\,\text{Log}\left[1+c\,x^2\right]-\frac{1}{4}\,b^2\,e\,x^2\,\text{Log}\left[1-c\,x^2\right]\,\text{Log}\left[1+c\,x^2\right]+\frac{1}{4}\,b^2\,d\,x\,\text{Log}\left[1+c\,x^2\right]^2+\frac{b^2\,e\,\left(1+c\,x^2\right)\,\text{Log}\left[1+c\,x^2\right]^2}{8\,c}+$$

$$\frac{a\,b\,e\,\text{Log}\left[1-c^2\,x^4\right]}{2\,c}-\frac{b^2\,e\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]}{4\,c}+\frac{b^2\,e\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{4\,c}+\frac{b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\sqrt{c}\,x}\right]}{\sqrt{c}}+$$

$$\frac{i\,b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2}{1-i\,\sqrt{c}\,x}\right]}{\sqrt{c}}-\frac{i\,b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{(1+i)\,\left(1-\sqrt{c}\,x\right)}{1-i\,\sqrt{c}\,x}\right]}{2\,\sqrt{c}}+\frac{i\,b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2}{1+i\,\sqrt{c}\,x}\right]}{\sqrt{c}}+\frac{b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2}{1+\sqrt{c}\,x}\right]}{\sqrt{c}}-$$

$$\frac{b^2\,d\,\text{PolyLog}\left[2,\,1+\frac{2\,\sqrt{c}\,\left(1-\sqrt{-c}\,x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\,\left(1+\sqrt{c}\,x\right)}\right]}{2\,\sqrt{c}}-\frac{b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2\,\sqrt{c}\,\left(1+\sqrt{-c}\,x\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\,\left(1+\sqrt{c}\,x\right)}\right]}{2\,\sqrt{c}}-\frac{i\,b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{(1-i)\,\left(1+\sqrt{c}\,x\right)}{1-i\,\sqrt{c}\,x}\right]}{2\,\sqrt{c}}$$

Problem 30: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{d+e\,x}\,dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{d+e\,x},\,x\right]$$

Result (type 8, 56 leaves, 2 steps):

$$2\,a\,b\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^2\right]}{d+e\,x},\,x\right]+b^2\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^2\right]^2}{d+e\,x},\,x\right]+\frac{a^2\,\text{Log}[d+e\,x]}{e}$$

Problem 31: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{(d+e\,x)^2}\,dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{(d+e\,x)^2},\,x\right]$$

Result (type 8, 202 leaves, 12 steps):

$$-\frac{a^2}{e\,(d+e\,x)}+\frac{2\,a\,b\,\sqrt{c}\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{c\,d^2+e^2}-\frac{2\,a\,b\,\sqrt{c}\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{c\,d^2-e^2}-\frac{2\,a\,b\,\text{ArcTanh}\left[c\,x^2\right]}{e\,(d+e\,x)}+$$
$$b^2\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^2\right]^2}{(d+e\,x)^2},\,x\right]+\frac{4\,a\,b\,c\,d\,e\,\text{Log}[d+e\,x]}{c^2\,d^4-e^4}-\frac{a\,b\,c\,d\,\text{Log}\left[1-c\,x^2\right]}{e\,\left(c\,d^2-e^2\right)}+\frac{a\,b\,c\,d\,\text{Log}\left[1+c\,x^2\right]}{e\,\left(c\,d^2+e^2\right)}$$

Problem 32: Result valid but suboptimal antiderivative.

$$\int(d+e\,x)^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\,dx$$

Optimal (type 3, 336 leaves, 24 steps):

$$-\frac{\sqrt{3}\,b\,d\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{2\,c^{2/3}}+\frac{\sqrt{3}\,b\,d\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{2\,c^{2/3}}+\frac{\sqrt{3}\,b\,d^2\,\text{ArcTan}\left[\frac{1+2\,c^{2/3}\,x^2}{\sqrt{3}}\right]}{2\,c^{1/3}}-$$
$$\frac{b\,d\,e\,\text{ArcTanh}\left[c^{1/3}\,x\right]}{c^{2/3}}+\frac{(d+e\,x)^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)}{3\,e}+\frac{b\,d^2\,\text{Log}\left[1-c^{2/3}\,x^2\right]}{2\,c^{1/3}}+\frac{b\,d\,e\,\text{Log}\left[1-c^{1/3}\,x+c^{2/3}\,x^2\right]}{4\,c^{2/3}}-$$
$$\frac{b\,d\,e\,\text{Log}\left[1+c^{1/3}\,x+c^{2/3}\,x^2\right]}{4\,c^{2/3}}+\frac{b\,\left(c\,d^3+e^3\right)\,\text{Log}\left[1-c\,x^3\right]}{6\,c\,e}-\frac{b\,\left(c\,d^3-e^3\right)\,\text{Log}\left[1+c\,x^3\right]}{6\,c\,e}-\frac{b\,d^2\,\text{Log}\left[1+c^{2/3}\,x^2+c^{4/3}\,x^4\right]}{4\,c^{1/3}}$$

Result (type 3, 332 leaves, 25 steps):

$$\frac{a\,(d+e\,x)^3}{3\,e}-\frac{\sqrt{3}\;b\,d\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{2\,c^{2/3}}+\frac{\sqrt{3}\;b\,d\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{2\,c^{2/3}}+\frac{\sqrt{3}\;b\,d^2\,\text{ArcTan}\left[\frac{1+2\,c^{2/3}\,x^2}{\sqrt{3}}\right]}{2\,c^{1/3}}-\frac{b\,d\,e\,\text{ArcTanh}\left[c^{1/3}\,x\right]}{c^{2/3}}+b\,d^2\,x\,\text{ArcTanh}\left[c\,x^3\right]+b\,d\,e\,x^2\,\text{ArcTanh}\left[c\,x^3\right]+\frac{1}{3}\,b\,e^2\,x^3\,\text{ArcTanh}\left[c\,x^3\right]+\frac{b\,d^2\,\text{Log}\left[1-c^{2/3}\,x^2\right]}{2\,c^{1/3}}+\frac{b\,d\,e\,\text{Log}\left[1-c^{1/3}\,x+c^{2/3}\,x^2\right]}{4\,c^{2/3}}-\frac{b\,d\,e\,\text{Log}\left[1+c^{1/3}\,x+c^{2/3}\,x^2\right]}{4\,c^{2/3}}-\frac{b\,d^2\,\text{Log}\left[1+c^{2/3}\,x^2+c^{4/3}\,x^4\right]}{4\,c^{1/3}}+\frac{b\,e^2\,\text{Log}\left[1-c^2\,x^6\right]}{6\,c}$$

Problem 33: Result optimal but 1 more steps used.

$$\int (d+e\,x)\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\,dx$$

Optimal (type 3, 285 leaves, 22 steps):

$$-\frac{\sqrt{3}\;b\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{4\,c^{2/3}}+\frac{\sqrt{3}\;b\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{4\,c^{2/3}}+\frac{\sqrt{3}\;b\,d\,\text{ArcTan}\left[\frac{1+2\,c^{2/3}\,x^2}{\sqrt{3}}\right]}{2\,c^{1/3}}-\frac{b\,e\,\text{ArcTanh}\left[c^{1/3}\,x\right]}{2\,c^{2/3}}-\frac{b\,d^2\,\text{ArcTanh}\left[c\,x^3\right]}{2\,e}+\frac{(d+e\,x)^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)}{2\,e}+\frac{b\,d\,\text{Log}\left[1-c^{2/3}\,x^2\right]}{2\,c^{1/3}}+\frac{b\,e\,\text{Log}\left[1-c^{1/3}\,x+c^{2/3}\,x^2\right]}{8\,c^{2/3}}-\frac{b\,e\,\text{Log}\left[1+c^{1/3}\,x+c^{2/3}\,x^2\right]}{8\,c^{2/3}}-\frac{b\,d\,\text{Log}\left[1+c^{2/3}\,x^2+c^{4/3}\,x^4\right]}{4\,c^{1/3}}$$

Result (type 3, 285 leaves, 23 steps):

$$-\frac{\sqrt{3}\;b\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{4\,c^{2/3}}+\frac{\sqrt{3}\;b\,e\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,c^{1/3}\,x}{\sqrt{3}}\right]}{4\,c^{2/3}}+\frac{\sqrt{3}\;b\,d\,\text{ArcTan}\left[\frac{1+2\,c^{2/3}\,x^2}{\sqrt{3}}\right]}{2\,c^{1/3}}-\frac{b\,e\,\text{ArcTanh}\left[c^{1/3}\,x\right]}{2\,c^{2/3}}-\frac{b\,d^2\,\text{ArcTanh}\left[c\,x^3\right]}{2\,e}+\frac{(d+e\,x)^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)}{2\,e}+\frac{b\,d\,\text{Log}\left[1-c^{2/3}\,x^2\right]}{2\,c^{1/3}}+\frac{b\,e\,\text{Log}\left[1-c^{1/3}\,x+c^{2/3}\,x^2\right]}{8\,c^{2/3}}-\frac{b\,e\,\text{Log}\left[1+c^{1/3}\,x+c^{2/3}\,x^2\right]}{8\,c^{2/3}}-\frac{b\,d\,\text{Log}\left[1+c^{2/3}\,x^2+c^{4/3}\,x^4\right]}{4\,c^{1/3}}$$

Problem 34: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,x^3\right]}{d+e\,x}\,dx$$

Optimal (type 4, 523 leaves, 25 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\,\text{Log}[d+e\,x]}{e}+\frac{b\,\text{Log}\left[\frac{e\left(1-c^{1/3}x\right)}{c^{1/3}d+e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{Log}\left[-\frac{e\left(1+c^{1/3}x\right)}{c^{1/3}d-e}\right]\,\text{Log}[d+e\,x]}{2\,e}+$$

$$\frac{b\,\text{Log}\left[-\frac{e\left((-1)^{1/3}+c^{1/3}x\right)}{c^{1/3}d-(-1)^{1/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{Log}\left[-\frac{e\left((-1)^{2/3}+c^{1/3}x\right)}{c^{1/3}d-(-1)^{2/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}+\frac{b\,\text{Log}\left[\frac{(-1)^{2/3}e\left(1+(-1)^{1/3}c^{1/3}x\right)}{c^{1/3}d+(-1)^{2/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}-$$

$$\frac{b\,\text{Log}\left[\frac{(-1)^{1/3}e\left(1+(-1)^{2/3}c^{1/3}x\right)}{c^{1/3}d+(-1)^{1/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d-e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d+e}\right]}{2\,e}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d-(-1)^{1/3}e}\right]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d+(-1)^{1/3}e}\right]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d-(-1)^{2/3}e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d+(-1)^{2/3}e}\right]}{2\,e}$$

Result (type 8, 30 leaves, 2 steps):

$$b\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^3\right]}{d+e\,x},x\right]+\frac{a\,\text{Log}[d+e\,x]}{e}$$

Problem 35: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^3\right]}{(d+e\,x)^2}\,dx$$

Optimal (type 3, 414 leaves, 19 steps):

$$-\frac{\sqrt{3}\,b\,c^{1/3}\,\text{ArcTan}\left[\frac{1-2\,c^{1/3}x}{\sqrt{3}}\right]}{2\left(c^{2/3}d^2+c^{1/3}d\,e+e^2\right)}-\frac{\sqrt{3}\,b\,c^{1/3}\left(c^{1/3}d+e\right)\,\text{ArcTan}\left[\frac{1+2\,c^{1/3}x}{\sqrt{3}}\right]}{2\left(c\,d^3+e^3\right)}-\frac{a+b\,\text{ArcTanh}\left[c\,x^3\right]}{e\,(d+e\,x)}+$$

$$\frac{b\,c^{1/3}\left(c^{1/3}d-e\right)\,\text{Log}\left[1-c^{1/3}x\right]}{2\left(c\,d^3+e^3\right)}+\frac{b\,c^{1/3}\left(c^{1/3}d+e\right)\,\text{Log}\left[1+c^{1/3}x\right]}{2\left(c\,d^3-e^3\right)}-\frac{3\,b\,c\,d^2\,e^2\,\text{Log}[d+e\,x]}{c^2\,d^6-e^6}-$$

$$\frac{b\,c^{1/3}\left(c^{1/3}d+e\right)\,\text{Log}\left[1-c^{1/3}x+c^{2/3}x^2\right]}{4\left(c\,d^3-e^3\right)}-\frac{b\,c^{1/3}\left(c^{1/3}d-e\right)\,\text{Log}\left[1+c^{1/3}x+c^{2/3}x^2\right]}{4\left(c\,d^3+e^3\right)}-\frac{b\,c\,d^2\,\text{Log}\left[1-c\,x^3\right]}{2\,e\left(c\,d^3+e^3\right)}+\frac{b\,c\,d^2\,\text{Log}\left[1+c\,x^3\right]}{2\,e\left(c\,d^3-e^3\right)}$$

Result (type 3, 414 leaves, 20 steps):

$$-\frac{\sqrt{3}\ b\ c^{1/3}\ \text{ArcTan}\left[\frac{1-2 c^{1/3} x}{\sqrt{3}}\right]}{2\ \left(c^{2/3} d^2+c^{1/3} d\ e+e^2\right)}-\frac{\sqrt{3}\ b\ c^{1/3}\ \left(c^{1/3} d+e\right)\ \text{ArcTan}\left[\frac{1+2 c^{1/3} x}{\sqrt{3}}\right]}{2\ \left(c\ d^3+e^3\right)}-\frac{a+b\ \text{ArcTanh}\left[c\ x^3\right]}{e\ (d+e\ x)}+$$
$$\frac{b\ c^{1/3}\ \left(c^{1/3} d-e\right)\ \text{Log}\left[1-c^{1/3} x\right]}{2\ \left(c\ d^3+e^3\right)}+\frac{b\ c^{1/3}\ \left(c^{1/3} d+e\right)\ \text{Log}\left[1+c^{1/3} x\right]}{2\ \left(c\ d^3-e^3\right)}-\frac{3\ b\ c\ d^2\ e^2\ \text{Log}[d+e\ x]}{c^2 d^6-e^6}-$$
$$\frac{b\ c^{1/3}\ \left(c^{1/3} d+e\right)\ \text{Log}\left[1-c^{1/3} x+c^{2/3} x^2\right]}{4\ \left(c\ d^3-e^3\right)}-\frac{b\ c^{1/3}\ \left(c^{1/3} d-e\right)\ \text{Log}\left[1+c^{1/3} x+c^{2/3} x^2\right]}{4\ \left(c\ d^3+e^3\right)}-\frac{b\ c\ d^2\ \text{Log}\left[1-c\ x^3\right]}{2\ e\ \left(c\ d^3+e^3\right)}+\frac{b\ c\ d^2\ \text{Log}\left[1+c\ x^3\right]}{2\ e\ \left(c\ d^3-e^3\right)}$$

Test results for the 538 problems in "7.3.4 u (a+b arctanh(c x))^p.m"

Problem 528: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)}{x^2}\, dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c\ e\ (a+b\ \text{ArcTanh}[c\ x])^2}{b}-\frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)}{x}+\frac{1}{2} b\ c\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)\ \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{2} b\ c\ e\ \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 94 leaves, 8 steps):

$$-\frac{c\ e\ (a+b\ \text{ArcTanh}[c\ x])^2}{b}+b\ c\ d\ \text{Log}[x]-\frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)}{x}-\frac{b\ c\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)^2}{4\ e}-\frac{1}{2} b\ c\ e\ \text{PolyLog}\left[2,c^2 x^2\right]$$

Problem 530: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)}{x^4}\, dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\frac{2\ c^2\ e\ (a+b\ \text{ArcTanh}[c\ x])}{3\ x}-\frac{c^3\ e\ (a+b\ \text{ArcTanh}[c\ x])^2}{3\ b}-b\ c^3\ e\ \text{Log}[x]+\frac{1}{3} b\ c^3\ e\ \text{Log}\left[1-c^2 x^2\right]-\frac{b\ c\ \left(1-c^2 x^2\right)\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)}{6\ x^2}-$$
$$\frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)}{3\ x^3}+\frac{1}{6} b\ c^3\ \left(d+e\ \text{Log}\left[1-c^2 x^2\right]\right)\ \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{6} b\ c^3\ e\ \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 191 leaves, 17 steps):

$$\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{3 x}-\frac{c^3 e (a+b \text{ArcTanh}[c x])^2}{3 b}+\frac{1}{3} b c^3 d \text{Log}[x]-b c^3 e \text{Log}[x]+\frac{1}{3} b c^3 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{6 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{3 x^3}-\frac{b c^3 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)^2}{12 e}-\frac{1}{6} b c^3 e \text{PolyLog}\left[2, c^2 x^2\right]$$

Problem 532: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x^6} \, dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{15 x^3}+\frac{2 c^4 e (a+b \text{ArcTanh}[c x])}{5 x}-\frac{c^5 e (a+b \text{ArcTanh}[c x])^2}{5 b}-\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3 \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}+\frac{1}{10} b c^5 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{10} b c^5 e \text{PolyLog}\left[2, \frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 250 leaves, 26 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{15 x^3}+\frac{2 c^4 e (a+b \text{ArcTanh}[c x])}{5 x}-\frac{c^5 e (a+b \text{ArcTanh}[c x])^2}{5 b}+\frac{1}{5} b c^5 d \text{Log}[x]-\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3 \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}-\frac{b c^5 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)^2}{20 e}-\frac{1}{10} b c^5 e \text{PolyLog}\left[2, c^2 x^2\right]$$

Test results for the 62 problems in "7.3.5 u (a+b arctanh(c+d x))^p.m"

Test results for the 1378 problems in "7.3.6 Exponentials of inverse hyperbolic tangent functions.m"

Problem 620: Result unnecessarily involves higher level functions.

$$\int e^{n \text{ArcTanh}[a x]} \left(c-\frac{c}{a x}\right)^2 \, dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{4\,c^2\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[2,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{a\,n}+\frac{2^{n/2}\,c^2\,(1-a\,x)^{2-\frac{n}{2}}\,\text{Hypergeometric2F1}\left[1-\frac{n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}\,(1-a\,x)\right]}{a\,(4-n)}$$

Result (type 6, 71 leaves, 3 steps):

$$\frac{2^{3-\frac{n}{2}}\,c^2\,(1+a\,x)^{\frac{2+n}{2}}\,\text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2}\,(-4+n),2,\frac{4+n}{2},\frac{1}{2}\,(1+a\,x),1+a\,x\right]}{a\,(2+n)}$$

Problem 621: Result valid but suboptimal antiderivative.

$$\int e^{n\,\text{ArcTanh}[a\,x]}\left(c-\frac{c}{a\,x}\right)dx$$

Optimal (type 5, 187 leaves, 6 steps):

$$\frac{c\,(1-a\,x)^{2-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{a\,(2-n)}-\frac{2\,c\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}\,(-2+n),\frac{n}{2},\frac{1+a\,x}{1-a\,x}\right]}{a\,(2-n)}+\frac{2^{n/2}\,c\,(1-n)\,(1-a\,x)^{2-\frac{n}{2}}\,\text{Hypergeometric2F1}\left[\frac{2-n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}\,(1-a\,x)\right]}{a\,(2-n)\,(4-n)}$$

Result (type 5, 184 leaves, 7 steps):

$$-\frac{2\,c\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,-\frac{n}{2},1-\frac{n}{2},\frac{1-a\,x}{1+a\,x}\right]}{a\,n}-\frac{2^{1+\frac{n}{2}}\,c\,(1-a\,x)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left[1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}\,(1-a\,x)\right]}{a\,(2-n)}+\frac{2^{1+\frac{n}{2}}\,c\,(1-a\,x)^{-n/2}\,\text{Hypergeometric2F1}\left[-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}\,(1-a\,x)\right]}{a\,n}$$

Problem 791: Result unnecessarily involves higher level functions.

$$\int e^{n\,\text{ArcTanh}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^2dx$$

Optimal (type 5, 331 leaves, 10 steps):

$$-\frac{4 c^2 (1-a x)^{3-\frac{n}{2}} (1+a x)^{\frac{1}{2}(-4+n)}}{a (4-n)}-\frac{c^2 (1-a x)^{3-\frac{n}{2}} (1+a x)^{\frac{1}{2}(-4+n)}}{3 a^4 x^3}-\frac{c^2 (10+n) (1-a x)^{3-\frac{n}{2}} (1+a x)^{\frac{1}{2}(-4+n)}}{6 a^3 x^2}-$$
$$\frac{c^2 (14+5 n+n^2) (1-a x)^{3-\frac{n}{2}} (1+a x)^{\frac{1}{2}(-4+n)}}{6 a^2 x}-\frac{c^2 n (10-n^2) (1-a x)^{2-\frac{n}{2}} (1+a x)^{\frac{1}{2}(-4+n)} \text{Hypergeometric2F1}\left[1,\frac{1}{2}(-4+n),\frac{1}{2}(-2+n),\frac{1+a x}{1-a x}\right]}{3 a (4-n)}+$$
$$\frac{2^{-1+\frac{n}{2}} c^2 n (1-a x)^{3-\frac{n}{2}} \text{Hypergeometric2F1}\left[\frac{4-n}{2},3-\frac{n}{2},4-\frac{n}{2},\frac{1}{2}(1-a x)\right]}{a (24-10 n+n^2)}$$

Result (type 6, 71 leaves, 3 steps):

$$\frac{2^{3-\frac{n}{2}} c^2 (1+a x)^{\frac{6+n}{2}} \text{AppellF1}\left[\frac{6+n}{2},\frac{1}{2}(-4+n),4,\frac{8+n}{2},\frac{1}{2}(1+a x),1+a x\right]}{a (6+n)}$$

Problem 792: Result unnecessarily involves higher level functions.

$$\int e^{n \text{ArcTanh}[a x]} \left(c-\frac{c}{a^2 x^2}\right) dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{4 c (1-a x)^{1-\frac{n}{2}} (1+a x)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-a x}{1+a x}\right]}{a (2-n)}-\frac{2^{1+\frac{n}{2}} c (1-a x)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left[1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-a x)\right]}{a (2-n)}$$

Result (type 6, 70 leaves, 3 steps):

$$-\frac{2^{2-\frac{n}{2}} c (1+a x)^{\frac{4+n}{2}} \text{AppellF1}\left[\frac{4+n}{2},\frac{1}{2}(-2+n),2,\frac{6+n}{2},\frac{1}{2}(1+a x),1+a x\right]}{a (4+n)}$$

Problem 795: Result unnecessarily involves higher level functions.

$$\int e^{n \text{ArcTanh}[a x]} \left(c-\frac{c}{a^2 x^2}\right)^{3/2} dx$$

Optimal (type 5, 430 leaves, 9 steps):

$$-\frac{\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x\,(1-a x)^{\frac{5-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}}{2\,(1-a^2 x^2)^{3/2}}-\frac{a\,(4+n)\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^2\,(1-a x)^{\frac{5-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}}{2\,(1-a^2 x^2)^{3/2}}-\frac{3 a^2\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1-a x)^{\frac{5-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}}{(3-n)\,(1-a^2 x^2)^{3/2}}-\frac{1}{(3-n)\,(1-a^2 x^2)^{3/2}}a^2\,(3-n^2)\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1-a x)^{\frac{3-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\frac{1+a x}{1-a x}\right]+\frac{2^{\frac{1}{2}(-1+n)}\,a^2\,n\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1-a x)^{\frac{5-n}{2}}\,\text{Hypergeometric2F1}\left[\frac{3-n}{2},\frac{5-n}{2},\frac{7-n}{2},\frac{1}{2}(1-a x)\right]}{(3-n)\,(5-n)\,(1-a^2 x^2)^{3/2}}$$

Result (type 6, 103 leaves, 3 steps):

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\,a^2\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1+a x)^{\frac{5+n}{2}}\,\text{AppellF1}\left[\frac{5+n}{2},\frac{1}{2}(-3+n),3,\frac{7+n}{2},\frac{1}{2}(1+a x),1+a x\right]}{(5+n)\,(1-a^2 x^2)^{3/2}}$$

Problem 796: Result valid but suboptimal antiderivative.

$$\int e^{n\,\text{ArcTanh}[a x]}\sqrt{c-\frac{c}{a^2 x^2}}\,dx$$

Optimal (type 5, 272 leaves, 6 steps):

$$-\frac{\sqrt{c-\frac{c}{a^2 x^2}}\;x\,(1-a x)^{\frac{3-n}{2}}\,(1+a x)^{\frac{1}{2}(-1+n)}}{(1-n)\sqrt{1-a^2 x^2}}+\frac{2\sqrt{c-\frac{c}{a^2 x^2}}\;x\,(1-a x)^{\frac{1-n}{2}}\,(1+a x)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a x}{1-a x}\right]}{(1-n)\sqrt{1-a^2 x^2}}+\frac{2^{\frac{1+n}{2}}\,n\sqrt{c-\frac{c}{a^2 x^2}}\;x\,(1-a x)^{\frac{3-n}{2}}\,\text{Hypergeometric2F1}\left[\frac{1-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-a x)\right]}{(3-4 n+n^2)\sqrt{1-a^2 x^2}}$$

Result (type 5, 302 leaves, 7 steps):

$$\frac{2\sqrt{c-\frac{c}{a^2 x^2}}\; x\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1-n),\frac{1-n}{2},\frac{1-a\,x}{1+a\,x}\right]}{(1+n)\sqrt{1-a^2 x^2}}-$$

$$\frac{2^{\frac{3+n}{2}}\sqrt{c-\frac{c}{a^2 x^2}}\; x\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,\text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n),\frac{1}{2}(-1-n),\frac{1-n}{2},\frac{1}{2}(1-a\,x)\right]}{(1+n)\sqrt{1-a^2 x^2}}+$$

$$\frac{2^{\frac{3+n}{2}}\sqrt{c-\frac{c}{a^2 x^2}}\; x\,(1-a\,x)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n),\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-a\,x)\right]}{(1-n)\sqrt{1-a^2 x^2}}$$

Problem 1316: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a\,x]}}{x\,(c-a^2 c\,x^2)}\,dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}}{c\,n}-\frac{2\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c\,n}$$

Result (type 5, 100 leaves, 3 steps):

$$\frac{(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}}{c\,n}-\frac{2\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-a\,x}{1+a\,x}\right]}{c\,(2-n)}$$

Problem 1317: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a\,x]}}{x^2\,(c-a^2 c\,x^2)}\,dx$$

Optimal (type 5, 123 leaves, 5 steps):

$$\frac{a\,(1+n)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}}{c\,n}-\frac{(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}}{c\,x}-\frac{2\,a\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c}$$

Result (type 5, 137 leaves, 5 steps):

$$\frac{a\,(1+n)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}}{c\,n}-\frac{(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}}{c\,x}-\frac{2\,a\,n\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-a\,x}{1+a\,x}\right]}{c\,(2-n)}$$

Problem 1323: Result valid but suboptimal antiderivative.

$$\int\frac{e^{n\,\text{ArcTanh}[a\,x]}}{x\,(c-a^2\,c\,x^2)^2}\,dx$$

Optimal (type 5, 190 leaves, 6 steps) :

$$\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{(4-n-n^2)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(4-n^2)}+$$
$$\frac{(4+n)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-\frac{2\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c^2\,n}$$

Result (type 5, 200 leaves, 6 steps) :

$$\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{(4-n-n^2)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(4-n^2)}+$$
$$\frac{(4+n)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-\frac{2\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-a\,x}{1+a\,x}\right]}{c^2\,(2-n)}$$

Problem 1324: Result valid but suboptimal antiderivative.

$$\int\frac{e^{n\,\text{ArcTanh}[a\,x]}}{x^2\,(c-a^2\,c\,x^2)^2}\,dx$$

Optimal (type 5, 239 leaves, 7 steps) :

$$\frac{a\,(3+n)\,(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,x}-\frac{a\,(6+4\,n-n^2-n^3)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(4-n^2)}+$$
$$\frac{a\,(6+4\,n+n^2)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-\frac{2\,a\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c^2}$$

Result (type 5, 253 leaves, 7 steps) :

$$\frac{a\,(3+n)\,(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,x}-\frac{a\,(6+4\,n-n^2-n^3)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(4-n^2)}+$$
$$\frac{a\,(6+4\,n+n^2)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-\frac{2\,a\,n\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-a\,x}{1+a\,x}\right]}{c^2\,(2-n)}$$

Problem 1331: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a\,x]}\,\sqrt{c-a^2\,c\,x^2}}{x}\,dx$$

Optimal (type 5, 269 leaves, 6 steps):

$$-\frac{(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{c-a^2\,c\,x^2}}{(1-n)\,\sqrt{1-a^2\,x^2}}+\frac{2\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{(1-n)\,\sqrt{1-a^2\,x^2}}+$$
$$\frac{2^{\frac{1+n}{2}}\,n\,(1-a\,x)^{\frac{3-n}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-a\,x)\right]}{(3-4\,n+n^2)\,\sqrt{1-a^2\,x^2}}$$

Result (type 5, 299 leaves, 7 steps):

$$\frac{2\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1+n}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1-n),\frac{1-n}{2},\frac{1-a\,x}{1+a\,x}\right]}{(1+n)\,\sqrt{1-a^2\,x^2}}-$$
$$\frac{2^{\frac{3+n}{2}}\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n),\frac{1}{2}(-1-n),\frac{1-n}{2},\frac{1}{2}(1-a\,x)\right]}{(1+n)\,\sqrt{1-a^2\,x^2}}+$$
$$\frac{2^{\frac{3+n}{2}}\,(1-a\,x)^{\frac{1-n}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n),\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-a\,x)\right]}{(1-n)\,\sqrt{1-a^2\,x^2}}$$

Problem 1332: Result unnecessarily involves higher level functions.

$$\int \frac{e^{n\,\text{ArcTanh}[a\,x]}\,\sqrt{c-a^2\,c\,x^2}}{x^2}\,dx$$

Optimal (type 5, 268 leaves, 6 steps):

$$-\frac{(1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1+n}{2}} \sqrt{c-a^2 c x^2}}{x \sqrt{1-a^2 x^2}}-\frac{2 a n (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2}(-1+n)} \sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}\left[1,\frac{1-n}{2},\frac{3-n}{2},\frac{1-a x}{1+a x}\right]}{(1-n) \sqrt{1-a^2 x^2}}+$$
$$\frac{2^{\frac{1+n}{2}} a (1-a x)^{\frac{1-n}{2}} \sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}\left[\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-a x)\right]}{(1-n) \sqrt{1-a^2 x^2}}$$

Result (type 6, 97 leaves, 3 steps):

$$\frac{2^{\frac{3}{2}-\frac{n}{2}} a (1+a x)^{\frac{3+n}{2}} \sqrt{c-a^2 c x^2}\ \text{AppellF1}\left[\frac{3+n}{2},\frac{1}{2}(-1+n),2,\frac{5+n}{2},\frac{1}{2}(1+a x),1+a x\right]}{(3+n) \sqrt{1-a^2 x^2}}$$

Problem 1345: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a x]}}{x \left(c-a^2 c x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 243 leaves, 6 steps):

$$\frac{(1-a x)^{\frac{1}{2}(-1-n)} (1+a x)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2 x^2}}{c (1+n) \sqrt{c-a^2 c x^2}}-\frac{(2+n) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2 x^2}}{c \left(1-n^2\right) \sqrt{c-a^2 c x^2}}+$$
$$\frac{2 (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a x}{1-a x}\right]}{c (1-n) \sqrt{c-a^2 c x^2}}$$

Result (type 5, 247 leaves, 6 steps):

$$\frac{(1-a x)^{\frac{1}{2}(-1-n)} (1+a x)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2 x^2}}{c (1+n) \sqrt{c-a^2 c x^2}}-\frac{(2+n) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2 x^2}}{c \left(1-n^2\right) \sqrt{c-a^2 c x^2}}-$$
$$\frac{2 (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a x}{1+a x}\right]}{c (3-n) \sqrt{c-a^2 c x^2}}$$

Problem 1346: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a x]}}{x^2 \left(c-a^2 c x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 321 leaves, 7 steps):

$$\frac{a\,(2+n)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,x\,\sqrt{c-a^2\,c\,x^2}}-$$
$$\frac{a\,(2+2\,n+n^2)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}+\frac{2\,a\,n\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c\,(1-n)\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 5, 325 leaves, 7 steps):

$$\frac{a\,(2+n)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,x\,\sqrt{c-a^2\,c\,x^2}}-$$
$$\frac{a\,(2+2\,n+n^2)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}-\frac{2\,a\,n\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a\,x}{1+a\,x}\right]}{c\,(3-n)\,\sqrt{c-a^2\,c\,x^2}}$$

Problem 1347: Result valid but suboptimal antiderivative.

$$\int\frac{e^{n\,\text{ArcTanh}[a\,x]}}{x^3\,(c-a^2\,c\,x^2)^{3/2}}\,dx$$

Optimal (type 5, 417 leaves, 8 steps):

$$\frac{a^2\,(3+2\,n+n^2)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,(1+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,x^2\,\sqrt{c-a^2\,c\,x^2}}-$$
$$\frac{a\,n\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,x\,\sqrt{c-a^2\,c\,x^2}}-\frac{a^2\,(6+5\,n+2\,n^2+n^3)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}+$$
$$\frac{a^2\,(3+n^2)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c\,(1-n)\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 5, 422 leaves, 8 steps):

$$\frac{a^2 \left(3+2 n+n^2\right) (1-a x)^{\frac{1}{2} (-1-n)} (1+a x)^{\frac{1}{2} (-1+n)} \sqrt{1-a^2 x^2}}{2 c (1+n) \sqrt{c-a^2 c x^2}}-\frac{(1-a x)^{\frac{1}{2} (-1-n)} (1+a x)^{\frac{1}{2} (-1+n)} \sqrt{1-a^2 x^2}}{2 c x^2 \sqrt{c-a^2 c x^2}}-$$

$$\frac{a n (1-a x)^{\frac{1}{2} (-1-n)} (1+a x)^{\frac{1}{2} (-1+n)} \sqrt{1-a^2 x^2}}{2 c x \sqrt{c-a^2 c x^2}}-\frac{a^2 \left(6+5 n+2 n^2+n^3\right) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2} (-1+n)} \sqrt{1-a^2 x^2}}{2 c \left(1-n^2\right) \sqrt{c-a^2 c x^2}}-$$

$$\frac{a^2 \left(3+n^2\right) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-a x}{1+a x}\right]}{c (3-n) \sqrt{c-a^2 c x^2}}$$

Problem 1352: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \text{ArcTanh}[a x]}}{x \left(c-a^2 c x^2\right)^{5/2}} \, dx$$

Optimal (type 5, 417 leaves, 8 steps):

$$\frac{(1-a x)^{\frac{1}{2} (-3-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) \sqrt{c-a^2 c x^2}}+\frac{(6+n) (1-a x)^{\frac{1}{2} (-1-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 (1+n) (3+n) \sqrt{c-a^2 c x^2}}-$$

$$\frac{\left(15+6 n+n^2\right) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) \left(1-n^2\right) \sqrt{c-a^2 c x^2}}+\frac{\left(18+7 n-2 n^2-n^3\right) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 \left(9-10 n^2+n^4\right) \sqrt{c-a^2 c x^2}}+$$

$$\frac{2 (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2} (-1+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1, \frac{1}{2} (-1+n), \frac{1+n}{2}, \frac{1+a x}{1-a x}\right]}{c^2 (1-n) \sqrt{c-a^2 c x^2}}$$

Result (type 5, 421 leaves, 8 steps):

$$\frac{(1-a x)^{\frac{1}{2} (-3-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) \sqrt{c-a^2 c x^2}}+\frac{(6+n) (1-a x)^{\frac{1}{2} (-1-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 (1+n) (3+n) \sqrt{c-a^2 c x^2}}-$$

$$\frac{\left(15+6 n+n^2\right) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) \left(1-n^2\right) \sqrt{c-a^2 c x^2}}+\frac{\left(18+7 n-2 n^2-n^3\right) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{c^2 \left(9-10 n^2+n^4\right) \sqrt{c-a^2 c x^2}}-$$

$$\frac{2 (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-a x}{1+a x}\right]}{c^2 (3-n) \sqrt{c-a^2 c x^2}}$$

Problem 1353: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \,\text{ArcTanh}[a\,x]}}{x^2\,\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 5, 507 leaves, 9 steps):

$$\frac{a\,(4+n)\,(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,x\,\sqrt{c-a^2\,c\,x^2}}+\frac{a\,\left(12+6\,n+n^2\right)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(1+n)\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{a\,\left(24+15\,n+6\,n^2+n^3\right)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(3+n)\,\left(1-n^2\right)\,\sqrt{c-a^2\,c\,x^2}}+\frac{a\,\left(24+18\,n+7\,n^2-2\,n^3-n^4\right)\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,\left(9-10\,n^2+n^4\right)\,\sqrt{c-a^2\,c\,x^2}}+\frac{2\,a\,n\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c^2\,(1-n)\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 5, 511 leaves, 9 steps):

$$\frac{a\,(4+n)\,(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,x\,\sqrt{c-a^2\,c\,x^2}}+\frac{a\,\left(12+6\,n+n^2\right)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(1+n)\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{a\,\left(24+15\,n+6\,n^2+n^3\right)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(3+n)\,\left(1-n^2\right)\,\sqrt{c-a^2\,c\,x^2}}+\frac{a\,\left(24+18\,n+7\,n^2-2\,n^3-n^4\right)\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,\left(9-10\,n^2+n^4\right)\,\sqrt{c-a^2\,c\,x^2}}-\frac{2\,a\,n\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a\,x}{1+a\,x}\right]}{c^2\,(3-n)\,\sqrt{c-a^2\,c\,x^2}}$$

Problem 1354: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \,\text{ArcTanh}[a\,x]}}{x^3\,\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 5, 623 leaves, 10 steps):

$$\frac{a^2 (5+4 n+n^2) (1-a x)^{\frac{1}{2}(-3-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (3+n) \sqrt{c-a^2 c x^2}}-\frac{(1-a x)^{\frac{1}{2}(-3-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 x^2 \sqrt{c-a^2 c x^2}}-$$
$$\frac{a n (1-a x)^{\frac{1}{2}(-3-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 x \sqrt{c-a^2 c x^2}}+\frac{a^2 (30+17 n+6 n^2+n^3) (1-a x)^{\frac{1}{2}(-1-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (1+n) (3+n) \sqrt{c-a^2 c x^2}}-$$
$$\frac{a^2 (75+54 n+20 n^2+6 n^3+n^4) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (3+n) (1-n^2) \sqrt{c-a^2 c x^2}}+\frac{a^2 (90+59 n+8 n^2+2 n^3-2 n^4-n^5) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (9-10 n^2+n^4) \sqrt{c-a^2 c x^2}}+$$
$$\frac{a^2 (5+n^2) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{1+a x}{1-a x}\right]}{c^2 (1-n) \sqrt{c-a^2 c x^2}}$$

Result (type 5, 628 leaves, 10 steps):

$$\frac{a^2 (5+4 n+n^2) (1-a x)^{\frac{1}{2}(-3-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (3+n) \sqrt{c-a^2 c x^2}}-\frac{(1-a x)^{\frac{1}{2}(-3-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 x^2 \sqrt{c-a^2 c x^2}}-$$
$$\frac{a n (1-a x)^{\frac{1}{2}(-3-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 x \sqrt{c-a^2 c x^2}}+\frac{a^2 (30+17 n+6 n^2+n^3) (1-a x)^{\frac{1}{2}(-1-n)} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (1+n) (3+n) \sqrt{c-a^2 c x^2}}-$$
$$\frac{a^2 (75+54 n+20 n^2+6 n^3+n^4) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (3+n) (1-n^2) \sqrt{c-a^2 c x^2}}+\frac{a^2 (90+59 n+8 n^2+2 n^3-2 n^4-n^5) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (9-10 n^2+n^4) \sqrt{c-a^2 c x^2}}-$$
$$\frac{a^2 (5+n^2) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-a x}{1+a x}\right]}{c^2 (3-n) \sqrt{c-a^2 c x^2}}$$

Test results for the 361 problems in "7.3.7 Inverse hyperbolic tangent functions.m"

Test results for the 300 problems in "7.4.1 Inverse hyperbolic cotangent functions.m"

Problem 76: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCoth}[a+b x]}{c+d x^2}\, dx$$

Optimal (type 4, 673 leaves, 15 steps):

$$\frac{\text{Log}\left[-\frac{1-a-b x}{a+b x}\right] \text{Log}\left[1+\frac{\left(b^2 c+a^2 d\right) (1-a-b x)}{\left(b^2 c-b \sqrt{-c} \sqrt{d}-(1-a) a d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}-\frac{\text{Log}\left[-\frac{1-a-b x}{a+b x}\right] \text{Log}\left[1+\frac{\left(b^2 c+a^2 d\right) (1-a-b x)}{\left(b^2 c+b \sqrt{-c} \sqrt{d}-(1-a) a d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}+$$
$$\frac{\text{Log}\left[\frac{1+a+b x}{a+b x}\right] \text{Log}\left[1-\frac{\left(b^2 c+a^2 d\right) (1+a+b x)}{\left(b^2 c-b \sqrt{-c} \sqrt{d}+a (1+a) d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}-\frac{\text{Log}\left[\frac{1+a+b x}{a+b x}\right] \text{Log}\left[1-\frac{\left(b^2 c+a^2 d\right) (1+a+b x)}{\left(b^2 c+b \sqrt{-c} \sqrt{d}+a (1+a) d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{\text{PolyLog}\left[2,-\frac{\left(b^2 c+a^2 d\right) (1-a-b x)}{\left(b^2 c-b \sqrt{-c} \sqrt{d}-(1-a) a d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}-$$
$$\frac{\text{PolyLog}\left[2,-\frac{\left(b^2 c+a^2 d\right) (1-a-b x)}{\left(b^2 c+b \sqrt{-c} \sqrt{d}-(1-a) a d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{\text{PolyLog}\left[2,\frac{\left(b^2 c+a^2 d\right) (1+a+b x)}{\left(b^2 c-b \sqrt{-c} \sqrt{d}+a (1+a) d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}-\frac{\text{PolyLog}\left[2,\frac{\left(b^2 c+a^2 d\right) (1+a+b x)}{\left(b^2 c+b \sqrt{-c} \sqrt{d}+a (1+a) d\right) (a+b x)}\right]}{4 \sqrt{-c} \sqrt{d}}$$

Result (type 4, 597 leaves, 37 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right] \left(\text{Log}[-1+a+b x]-\text{Log}\left[-\frac{1-a-b x}{a+b x}\right]-\text{Log}[a+b x]\right)}{2 \sqrt{c} \sqrt{d}}+$$
$$\frac{\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right] \left(\text{Log}[a+b x]-\text{Log}[1+a+b x]+\text{Log}\left[\frac{1+a+b x}{a+b x}\right]\right)}{2 \sqrt{c} \sqrt{d}}-\frac{\text{Log}[-1+a+b x] \text{Log}\left[\frac{b \left(\sqrt{-c}-\sqrt{d}\, x\right)}{b \sqrt{-c}-(1-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+$$
$$\frac{\text{Log}[1+a+b x] \text{Log}\left[\frac{b \left(\sqrt{-c}-\sqrt{d}\, x\right)}{b \sqrt{-c}+(1+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{\text{Log}[-1+a+b x] \text{Log}\left[\frac{b \left(\sqrt{-c}+\sqrt{d}\, x\right)}{b \sqrt{-c}+(1-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}-\frac{\text{Log}[1+a+b x] \text{Log}\left[\frac{b \left(\sqrt{-c}+\sqrt{d}\, x\right)}{b \sqrt{-c}-(1+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}-$$
$$\frac{\text{PolyLog}\left[2,-\frac{\sqrt{d}\, (1-a-b x)}{b \sqrt{-c}-(1-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{\text{PolyLog}\left[2,\frac{\sqrt{d}\, (1-a-b x)}{b \sqrt{-c}+(1-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}-\frac{\text{PolyLog}\left[2,-\frac{\sqrt{d}\, (1+a+b x)}{b \sqrt{-c}-(1+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{\text{PolyLog}\left[2,\frac{\sqrt{d}\, (1+a+b x)}{b \sqrt{-c}+(1+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}$$

Problem 275: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \text{ArcCoth}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x^2} \, dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c e (a+b \text{ArcCoth}[c x])^2}{b}-\frac{(a+b \text{ArcCoth}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x}+\frac{1}{2} b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{2} b c e \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 94 leaves, 8 steps):

$$-\frac{c e (a+b \text{ArcCoth}[c x])^2}{b}+b c d \text{Log}[x]-\frac{(a+b \text{ArcCoth}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x}-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)^2}{4 e}-\frac{1}{2} b c e \text{PolyLog}\left[2,c^2 x^2\right]$$

Problem 276: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{x^4}\,dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\frac{2 c^2 e\,(a+b\,\text{ArcCoth}[c\,x])}{3 x}-\frac{c^3 e\,(a+b\,\text{ArcCoth}[c\,x])^2}{3 b}-b c^3 e\,\text{Log}[x]+\frac{1}{3} b c^3 e\,\text{Log}[1-c^2 x^2]-\frac{b c\,(1-c^2 x^2)\,(d+e\,\text{Log}[1-c^2 x^2])}{6 x^2}-\frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{3 x^3}+\frac{1}{6} b c^3\,(d+e\,\text{Log}[1-c^2 x^2])\,\text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{6} b c^3 e\,\text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 191 leaves, 17 steps):

$$\frac{2 c^2 e\,(a+b\,\text{ArcCoth}[c\,x])}{3 x}-\frac{c^3 e\,(a+b\,\text{ArcCoth}[c\,x])^2}{3 b}+\frac{1}{3} b c^3 d\,\text{Log}[x]-b c^3 e\,\text{Log}[x]+\frac{1}{3} b c^3 e\,\text{Log}[1-c^2 x^2]-\frac{b c\,(1-c^2 x^2)\,(d+e\,\text{Log}[1-c^2 x^2])}{6 x^2}-\frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{3 x^3}-\frac{b c^3\,(d+e\,\text{Log}[1-c^2 x^2])^2}{12 e}-\frac{1}{6} b c^3 e\,\text{PolyLog}[2,c^2 x^2]$$

Problem 277: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{x^6}\,dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e\,(a+b\,\text{ArcCoth}[c\,x])}{15 x^3}+\frac{2 c^4 e\,(a+b\,\text{ArcCoth}[c\,x])}{5 x}-\frac{c^5 e\,(a+b\,\text{ArcCoth}[c\,x])^2}{5 b}-\frac{5}{6} b c^5 e\,\text{Log}[x]+\frac{19}{60} b c^5 e\,\text{Log}[1-c^2 x^2]-\frac{b c\,(d+e\,\text{Log}[1-c^2 x^2])}{20 x^4}-\frac{b c^3\,(1-c^2 x^2)\,(d+e\,\text{Log}[1-c^2 x^2])}{10 x^2}-\frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{5 x^5}+\frac{1}{10} b c^5\,(d+e\,\text{Log}[1-c^2 x^2])\,\text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{10} b c^5 e\,\text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 250 leaves, 26 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e\,(a+b\,\text{ArcCoth}[c\,x])}{15 x^3}+\frac{2 c^4 e\,(a+b\,\text{ArcCoth}[c\,x])}{5 x}-\frac{c^5 e\,(a+b\,\text{ArcCoth}[c\,x])^2}{5 b}+\frac{1}{5} b c^5 d\,\text{Log}[x]-\frac{5}{6} b c^5 e\,\text{Log}[x]+\frac{19}{60} b c^5 e\,\text{Log}\left[1-c^2 x^2\right]-\frac{b c\,\left(d+e\,\text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3\,\left(1-c^2 x^2\right)\left(d+e\,\text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{(a+b\,\text{ArcCoth}[c\,x])\left(d+e\,\text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}-\frac{b c^5\left(d+e\,\text{Log}\left[1-c^2 x^2\right]\right)^2}{20 e}-\frac{1}{10} b c^5 e\,\text{PolyLog}\left[2, c^2 x^2\right]$$

Test results for the 935 problems in "7.4.2 Exponentials of inverse hyperbolic cotangent functions.m"

Problem 542: Result unnecessarily involves higher level functions.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a x}\right) dx$$

Optimal (type 5, 185 leaves, 5 steps):

$$c\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{n/2} x-\frac{2 c\,(1-n)\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2}\text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a n}-\frac{2^{n/2} c\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left[1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2 a}\right]}{a\,(2-n)}$$

Result (type 6, 81 leaves, 2 steps):

$$-\frac{2^{2-\frac{n}{2}} c\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}}\text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2}(-2+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a\,(2+n)}$$

Problem 751: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}\,x^3}{\left(c-a^2 c x^2\right)^{3/2}} dx$$

Optimal (type 5, 359 leaves, 7 steps):

$$-\frac{(2+n)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1+n)\left(c-a^2 c x^2\right)^{3/2}}+\frac{\left(2+2 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1-n)(1+n)\left(c-a^2 c x^2\right)^{3/2}}+$$

$$\frac{\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^4}{\left(c-a^2 c x^2\right)^{3/2}}-\frac{2 n\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3 \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a(1-n)\left(c-a^2 c x^2\right)^{3/2}}$$

Result (type 5, 363 leaves, 7 steps):

$$-\frac{(2+n)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1+n)\left(c-a^2 c x^2\right)^{3/2}}+\frac{\left(2+2 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1-n)(1+n)\left(c-a^2 c x^2\right)^{3/2}}+$$

$$\frac{\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^4}{\left(c-a^2 c x^2\right)^{3/2}}+\frac{2 n\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^3 \text{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a(3-n)\left(c-a^2 c x^2\right)^{3/2}}$$

Problem 756: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \text{ArcCoth}[a x]} x^4}{\left(c-a^2 c x^2\right)^{5/2}} \, dx$$

Optimal (type 5, 463 leaves, 8 steps):

$$-\frac{\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2 c x^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}+$$

$$\frac{\left(15+6 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1-n)(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}-\frac{\left(18+7 n-2 n^2-n^3\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{\left(9-10 n^2+n^4\right)\left(c-a^2 c x^2\right)^{5/2}}-$$

$$\frac{2\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^5 \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{(1-n)\left(c-a^2 c x^2\right)^{5/2}}$$

Result (type 5, 467 leaves, 8 steps):

$$-\frac{\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2 c x^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}+$$

$$\frac{\left(15+6 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1-n)(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}-\frac{\left(18+7 n-2 n^2-n^3\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{\left(9-10 n^2+n^4\right)\left(c-a^2 c x^2\right)^{5/2}}+$$

$$\frac{2\left(1-\frac{1}{a^2 x^2}\right)^{5/2}\left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5 \text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{(3-n)\left(c-a^2 c x^2\right)^{5/2}}$$

Problem 928: Result unnecessarily involves higher level functions.

$$\int e^{n \text{ArcCoth}[a x]}\left(c-\frac{c}{a^2 x^2}\right) dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$\frac{4 c\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a(2-n)}-\frac{2^{1+\frac{n}{2}} c\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left[1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2 a}\right]}{a(2-n)}$$

Result (type 6, 81 leaves, 2 steps):

$$-\frac{2^{2-\frac{n}{2}} c\left(1+\frac{1}{a x}\right)^{\frac{4+n}{2}} \text{AppellF1}\left[\frac{4+n}{2},\frac{1}{2}(-2+n),2,\frac{6+n}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a(4+n)}$$

Problem 929: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \text{ArcCoth}[a x]}}{c-\frac{c}{a^2 x^2}} dx$$

Optimal (type 5, 150 leaves, 5 steps):

$$-\frac{(1+n)\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2}}{a c n}+\frac{\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2} x}{c}+\frac{2\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2} \text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a c}$$

Result (type 5, 164 leaves, 5 steps):

$$-\frac{(1+n)\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2}}{a c n}+\frac{\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2} x}{c}+\frac{2 n\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a c (2-n)}$$

Problem 930: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcCoth}[a x]}}{\left(c-\frac{c}{a^2 x^2}\right)^2} \, dx$$

Optimal (type 5, 289 leaves, 7 steps):

$$-\frac{(3+n)\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2+n)}+\frac{\left(6+4 n-n^2-n^3\right)\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2-n) n (2+n)}-\frac{\left(6+4 n+n^2\right)\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 n (2+n)}+$$

$$\frac{\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} x}{c^2}+\frac{2\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2} \text{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a c^2}$$

Result (type 5, 303 leaves, 7 steps):

$$-\frac{(3+n)\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2+n)}+\frac{\left(6+4 n-n^2-n^3\right)\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2-n) n (2+n)}-\frac{\left(6+4 n+n^2\right)\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 n (2+n)}+$$

$$\frac{\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} x}{c^2}+\frac{2 n\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a c^2 (2-n)}$$

Problem 931: Result unnecessarily involves higher level functions.

$$\int e^{n \operatorname{ArcCoth}[a x]} \sqrt{c-\frac{c}{a^2 x^2}} \, dx$$

Optimal (type 5, 295 leaves, 6 steps):

$$\frac{\sqrt{c-\frac{c}{a^2 x^2}}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1+n}{2}} x}{\sqrt{1-\frac{1}{a^2 x^2}}}+\frac{2 n \sqrt{c-\frac{c}{a^2 x^2}}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} \text{Hypergeometric2F1}\left[1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a(1-n)\sqrt{1-\frac{1}{a^2 x^2}}}-$$

$$\frac{2^{\frac{1+n}{2}}\sqrt{c-\frac{c}{a^2 x^2}}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}} \text{Hypergeometric2F1}\left[\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2 a}\right]}{a(1-n)\sqrt{1-\frac{1}{a^2 x^2}}}$$

Result (type 6, 111 leaves, 3 steps):

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{a^2 x^2}}\left(1+\frac{1}{a x}\right)^{\frac{3+n}{2}} \text{AppellF1}\left[\frac{3+n}{2},\frac{1}{2}(-1+n),2,\frac{5+n}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a(3+n)\sqrt{1-\frac{1}{a^2 x^2}}}$$

Test results for the 190 problems in "7.5.1 u (a+b arcsech(c x))^n.m"

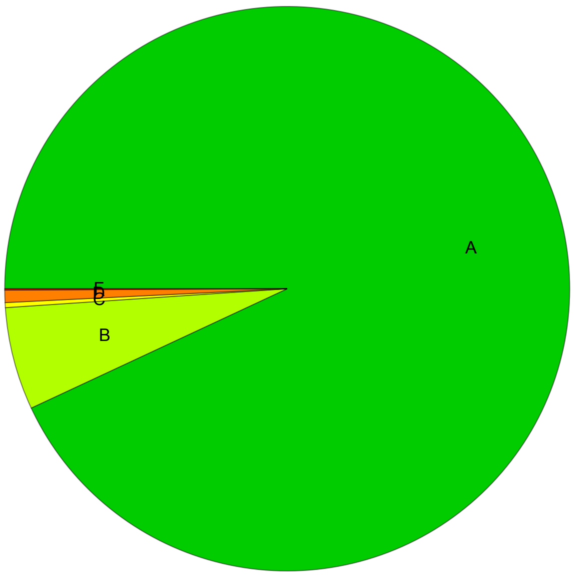
Test results for the 100 problems in "7.5.2 Inverse hyperbolic secant functions.m"

Test results for the 178 problems in "7.6.1 u (a+b arccsch(c x))^n.m"

Test results for the 71 problems in "7.6.2 Inverse hyperbolic cosecant functions.m"

Summary of Integration Test Results

6650 integration problems



A - 6193 optimal antiderivatives

B - 391 valid but suboptimal antiderivatives

C - 19 unnecessarily complex antiderivatives

D - 47 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives